Advanced Photon Source

Transverse and Temporal Characteristics of a High-Gain FEL in the Saturation Regime

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Introduction

- Many current high-gain FEL experiments reach saturation (HGHG, LEUTL, VISA…)

- Future light sources based on high-gain FELs tend to operate at or after saturation for maximum power and/or high stability

- In the exponential growth regime, gain (e-folding) length
  Transverse: gain guiding of a single mode
  Temporal: gain narrowing of the SASE spectrum

- Important to understand saturation behaviors
  Transverse: refractive guiding
  Temporal: sideband instability

- We study these effects self-consistently, compare theory with GINGER simulations, and discuss implications to experiments

Refractive guiding

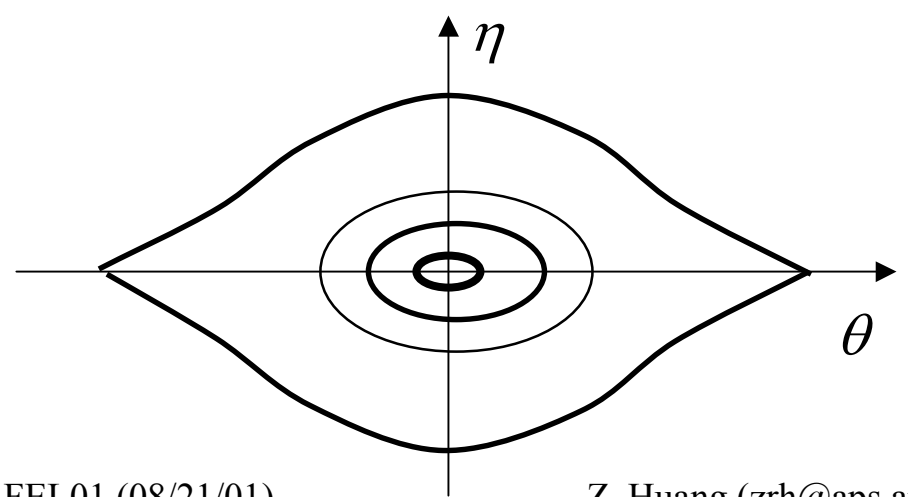
- Bunched beam generates an effective index of refraction (>1) to guide the radiation (Scharlemann et al., 1985, fiber analog)

- Unlike fibers, nonlinear interaction between e-beam and radiation
  ➔ self-consistency ➔ Maxwell-Vlasov approach

- After saturation, power stays roughly constant, but phase still advance

$$A(\mathbf{x})\mathrm{e}^{i\kappa\bar{z}}\mathrm{e}^{i\,k_r\,(z-ct)}$$

- Electrons are trapped by the ponderomotive potential (bucket)



electron distribution

$$F(H_0) \propto \mathrm{e}^{-\alpha H_0}$$

Index of refraction

- Maxwell equation becomes the mode equation

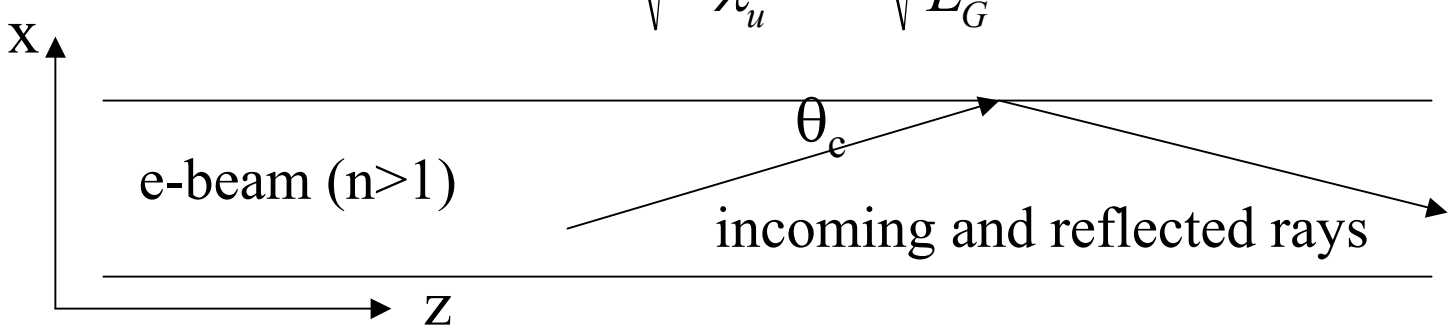
$$\boxed{(\kappa - \overline{\nabla}^2/2)A(\mathbf{x}) \approx \alpha U(\mathbf{x})A(\mathbf{x})}$$

Propagation constant $\beta = k_r + 2\rho k_u \kappa$

Index of refraction $n = 1 + \frac{2\rho k_u}{k_r}\alpha U(\mathbf{x})$ ← e-beam profile

- Use the Snell's law, the critical angle for total internal reflection is

$$\theta_c = \sqrt{2(\delta n)} \sim \sqrt{\frac{4\lambda_r \rho}{\lambda_u}} \sim \sqrt{\frac{\lambda_r}{L_G}} \quad \text{saturation } \alpha \sim 1$$

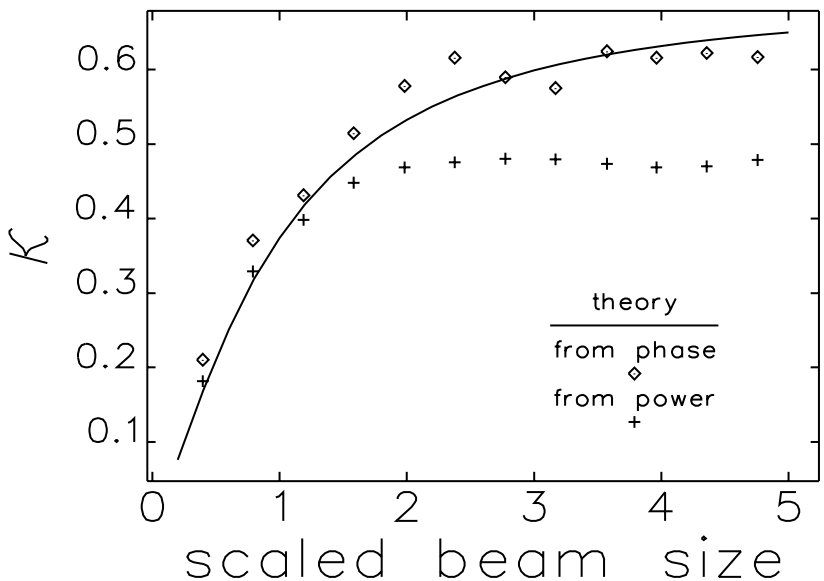


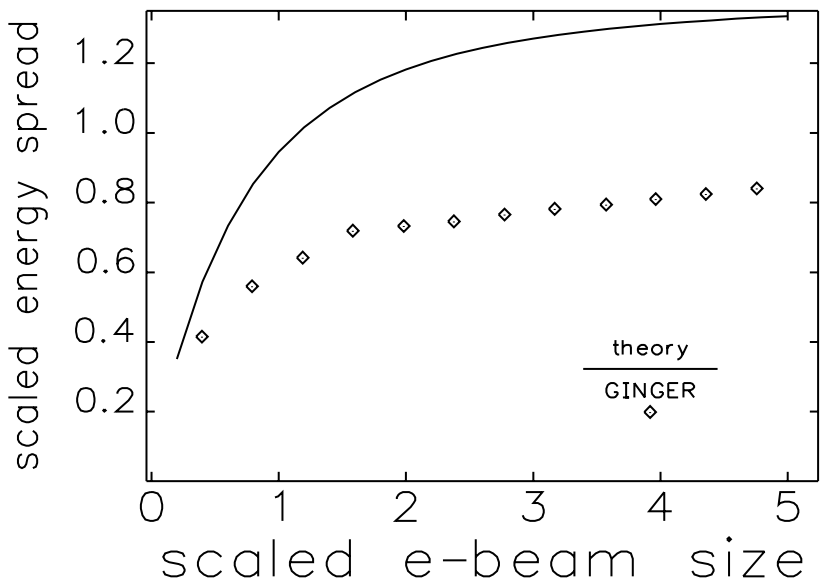
Equilibrium state

- Energy conservation and another constant of motion constraint the equilibrium state (Gluckstern,Krinsky,Okamoto 1993):

$$\kappa = \frac{1}{2\rho k_u}\frac{d\phi}{dz} = \frac{P_s}{\rho P_{\text{beam}}} \text{ (equilibrium power)}$$

$$\sqrt{\alpha^{-1} + \kappa^2} = \frac{\sigma_\delta}{\rho} \text{ (equilibrium energy spread)}$$



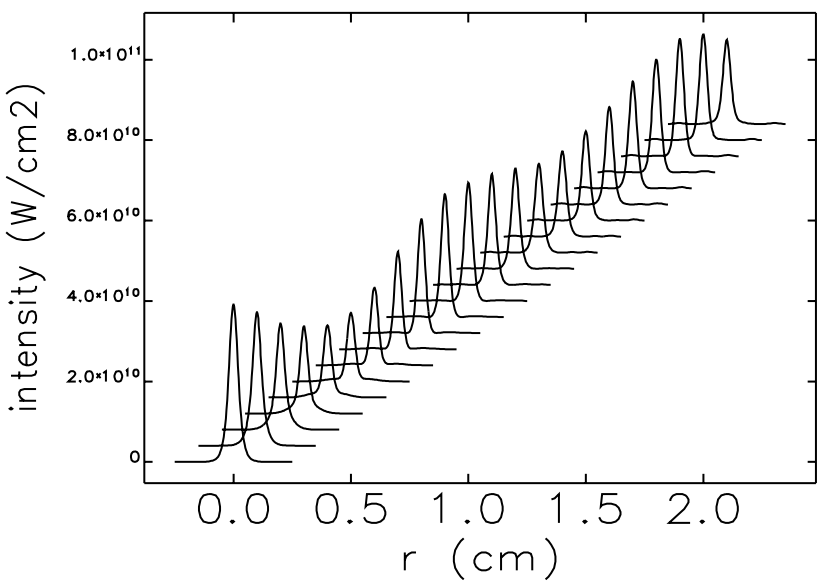


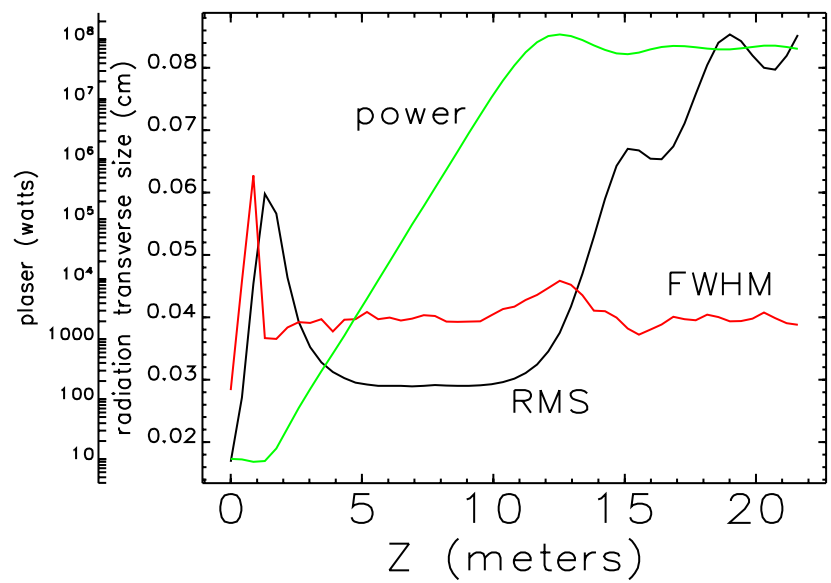
Transverse profile

- Gaussian-like guided mode that carries a fixed amount of power

- After an initial power overshoot at the saturation point, the excess radiation power diffracts out the e-beam ➔ increase rms size, beam-radiation system relax to the equilibrium state➔constant FWHM

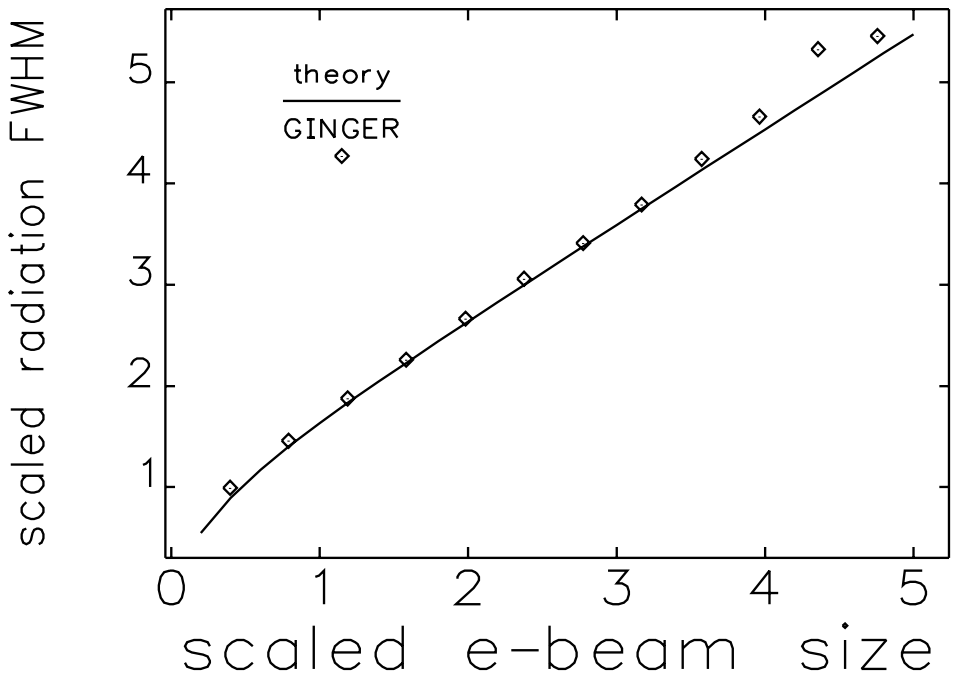
- GINGER simulation of the LEUTL FEL mode evolution





Guided mode size

- Zero emittance and no initial energy spread



- Initial energy spread can be included in this analysis
- For current FELs (LEUTL…), emittance < $\lambda/4\pi$, adaquate analysis
- For future x-ray FELs (LCLS), emittance > $\lambda/4\pi$, must be included
  simulations show guided radiation with complex transverse evolution

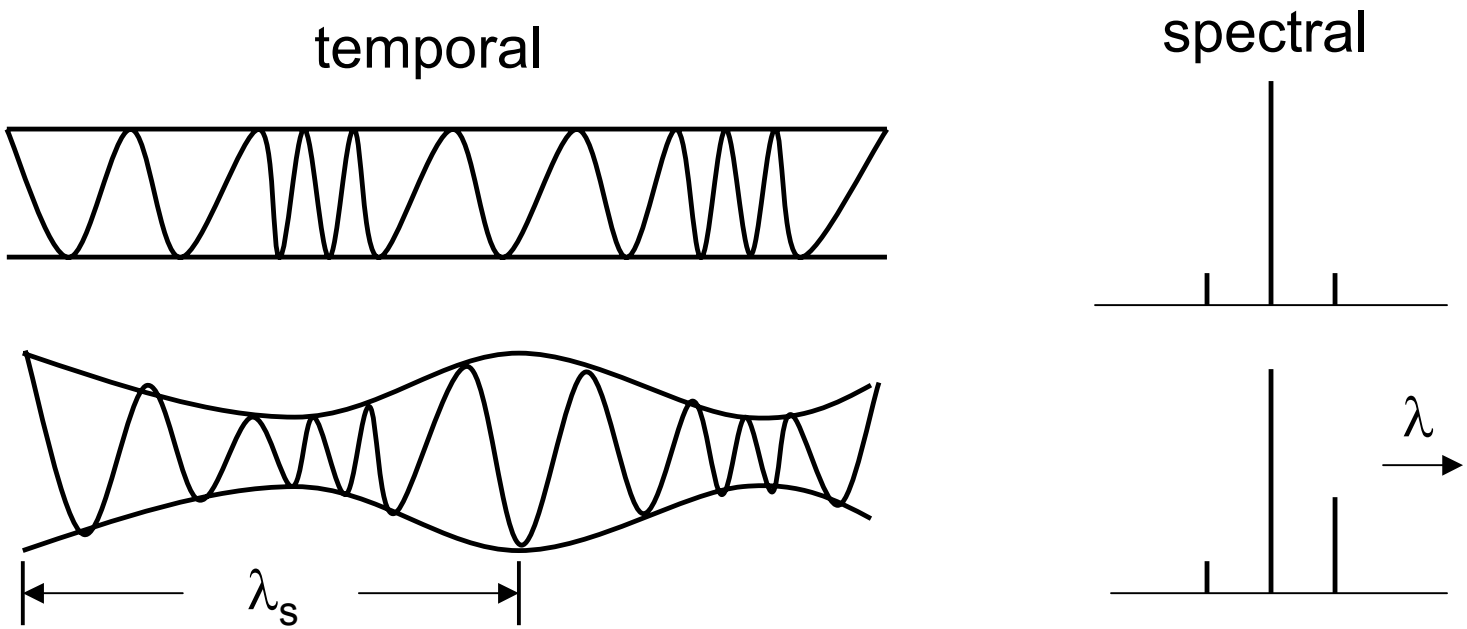
Sideband instability

- Trapped electrons execute synchrotron oscillation in the bucket

synchrotron frequency $\Omega_0 = 2\rho\omega_r\sqrt{2A}$

- A wave that is shifted by Ω_0 can interact resonantly with the syn. motion ➔ sideband instability (Kroll, Morton, Rosenbluth 1981)

- A physical picture of sideband evolution is given by Warren, 1986



Coupled sideband modes

- Linearize Vlasov-Maxwell equations around the equilibrium state

- Coupled equations for lower and upper sidebands (Elgin, 1991)

1-D case
$$\begin{cases} \dfrac{da_+}{dz} = C_0 a_+ + C_2 a_-^*, \\ \dfrac{da_-^*}{dz} = -C_0 a_-^* - C_2 a_+. \end{cases}$$

- asymmetric eigenmode: a mixture of lower and upper sidebands

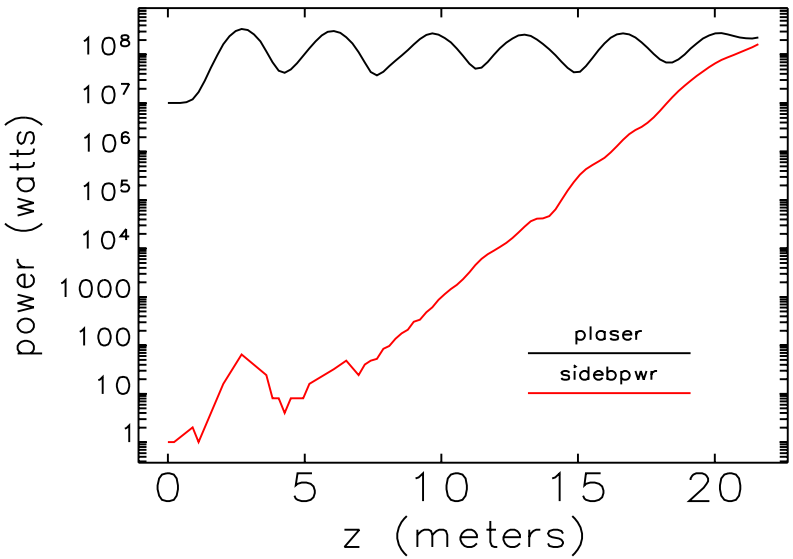
$$a_+ \approx \frac{2C_0}{C_2} a_- >> a_- \text{ for } C_0 >> C_2$$

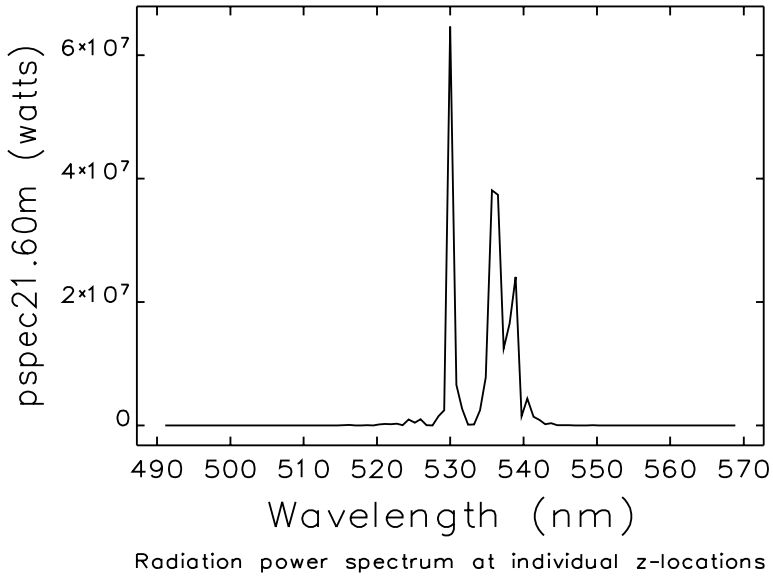
- Symmetric sideband growth rate

$$\boxed{\Gamma \approx C_0 = \frac{1.7\lambda_u}{4\pi\rho} = 3L_G^{1\mathrm{D}}}$$

Simulation and SASE

- GINGER amplifier simulations agree with theory within 10%.





- For SASE, linewidth at saturation ~ ρ < sideband location ~2ρ
  sideband physics qualitatively the same but more nonlinear

➔ slow power increase, spectrum has redshifted spikes and broaden)

- LEUTL FEL has glimpse of sideband dynamics (Sajaev, Lumpkin)

Conclusions

- Signature of high-gain FEL saturation:

  ➔deviation of exponential gain and reduced fluctuation
  ➔increase of e-beam energy spread
  ➔strong nonlinear harmonic radiation

  ➔Radiation characteristics
  transverse: rms size grow (unguided radiation)
  guided radiation remain confined
  temporal: correlated phase and amplitude
  spectrum red-shifts and broader