Bunch Compression

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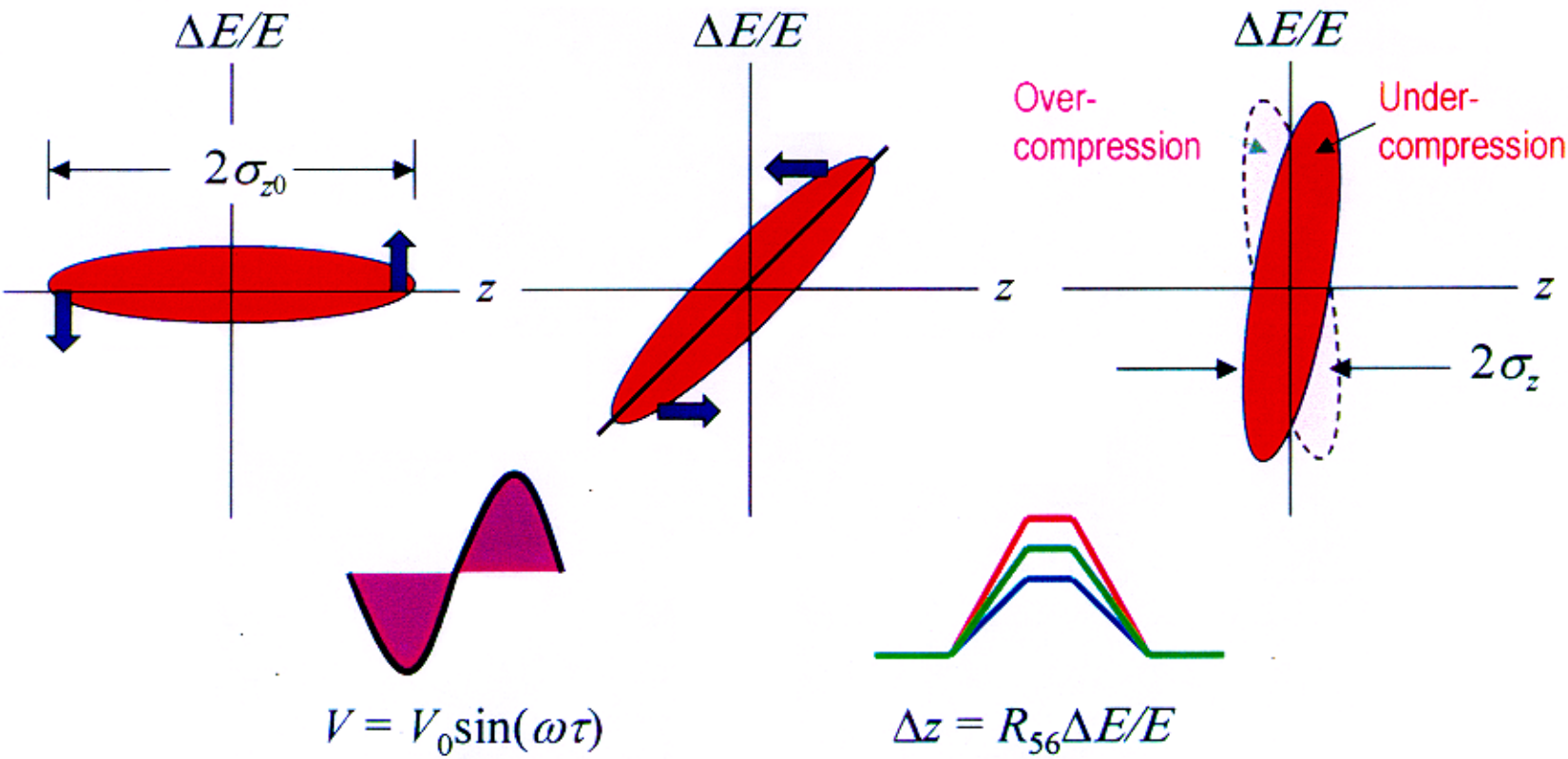
17th Advanced Beam Dynamics Workshop on
Future Light Sources

- Introduction
- 2nd order effects
- Wakefields
- CSR and ISR limitations
- Low-charge LCLS option
- NLC optimization example

Magnetic Bunch Compression ($\gamma >> 1$)



$V = V_0 \sin(\omega\tau)$

$\Delta z = R_{56}\Delta E/E$

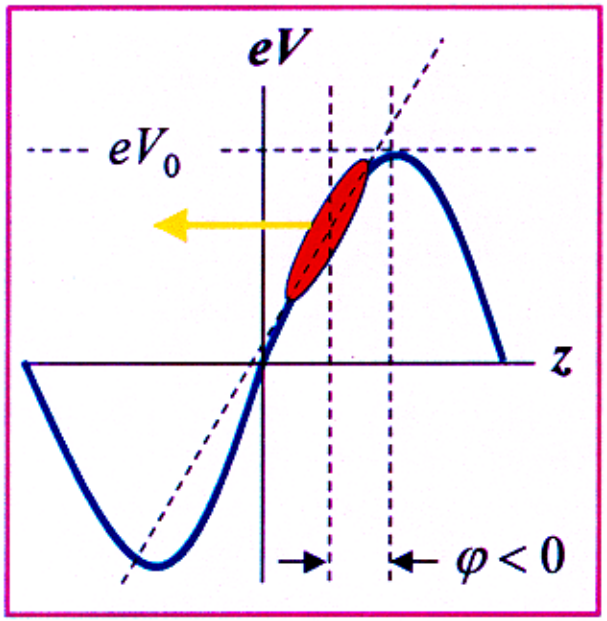
RF Accelerating Voltage

Path Length-Energy Dependent Beamline

Compressor Design Issues

- Types of compressors (chicane, wiggler, arc, …)
- Linearity (RF and optics)
- Pulse-to-pulse jitter sensitivity
- RF phasing and acceleration efficiency
- Longitudinal wakefields (geometric, resistive, …)
- Transverse wakefield minimization with short bunches
- Energy spread limitations (final and intermediate)
- Synchrotron radiation (emittance and energy spread)
- Space charge effects (emittance and energy spread)
- Reasonable dispersion (beam size) in compressor
- Cost optimization of compressor system (RF & magnets)
- What color to paint magnets...

Single-Stage Bunch Compression



$$E(z) = E_0 + eV_0 \cos(\varphi + 2\pi z/\lambda)$$

$$\delta \equiv \frac{\Delta E}{E} \approx \ldots$$

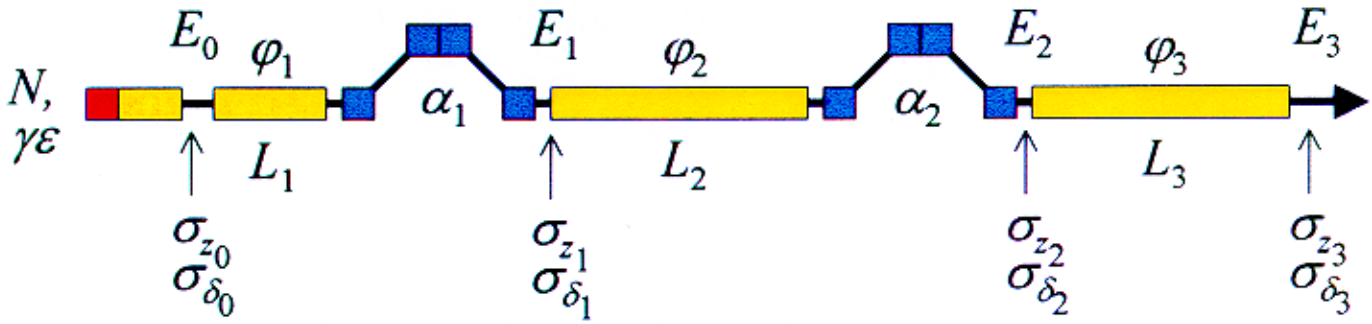
$$\delta_0 \frac{E_0}{E} + \left(1 - \frac{E_0}{E}\right)\left[\frac{\cos(\varphi + \Delta\varphi) - (2\pi z/\lambda)\sin(\varphi + \Delta\varphi)}{\cos(\varphi)} - 1\right]$$

$$k(\varphi) \equiv \frac{\partial \delta}{\partial z} = -\frac{2\pi}{\lambda}\left(1 - \frac{E_0}{E}\right)\frac{\sin(\varphi + \Delta\varphi)}{\cos(\varphi)}$$

$$\sigma_z = \sqrt{(1 + kR_{56})^2 \sigma_{z0}^2 + R_{56}^2 \sigma_{\delta 0}^2 E_0^2/E^2} \quad , \quad \sigma_\delta = \sqrt{k^2 \sigma_{z0}^2 + \sigma_{\delta 0}^2 E_0^2/E^2}$$

$$\frac{\Delta\sigma_z}{\sigma_z} \approx -\left(\frac{\sigma_{z0}}{\sigma_z} \mp 1\right)\Delta\varphi \cot(\varphi) \quad \Rightarrow \quad \frac{\sigma_{z0}}{\sigma_z} = 40: \quad 25\% \text{ jitter} / 0.1 \text{ psec} \quad @ -15^\circ$$

Two-Stage Bunch Compression



Even linear equations are messy - wakes require tracking calculations

$$\begin{pmatrix} z_3 \\ \delta_3 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ k_3 & E_2/E_3 \end{bmatrix} \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ k_2 & E_1/E_2 \end{bmatrix} \begin{bmatrix} 1 & \alpha_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ k_1 & E_0/E_1 \end{bmatrix} \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}$$

$$\sigma_{z_3}^2 \approx \left[(1+\alpha_1 k_1)(1+\alpha_2 k_2) + \alpha_2 k_1 E_1/E_2\right]^2 \sigma_{z_0}^2 + \left[\alpha_1(1+\alpha_2 k_2)E_0/E_1 + \alpha_2 E_0/E_2\right]^2 \sigma_{\delta_0}^2$$

$$\sigma_{\delta_3}^2 \approx \left\{k_3\left[(1+\alpha_1 k_1)(1+\alpha_2 k_2) + \alpha_2 k_1 E_1/E_2\right] + \left[k_2(1+\alpha_1 k_1) + k_1 E_1/E_2\right]E_2/E_3\right\}^2 \sigma_{z_0}^2 + \left\{k_3\left[\alpha_1(1+\alpha_2 k_2)E_0/E_1 + \alpha_2 E_0/E_2\right] + \left[k_2\alpha_1 E_0/E_1 + E_0/E_2\right]E_2/E_3\right\}^2 \sigma_{\delta_0}^2$$

2nd Order Compression Limitations

For chicane or wiggler (any 'non-focusing' compressor), the path length...

$$\Delta s \sim \left(\frac{\theta_0}{1+\delta}\right)^2 = \theta_0^2\left(1-2\delta+3\delta^2-\cdots\right) = \Delta s_0 + R_{56}\delta + T_{566}\delta^2 + \cdots$$

$$\boxed{T_{566} \approx -\frac{3}{2}R_{56} \equiv rR_{56}}$$

$$z \approx z_0 + R_{56}\delta(1+r\delta)$$

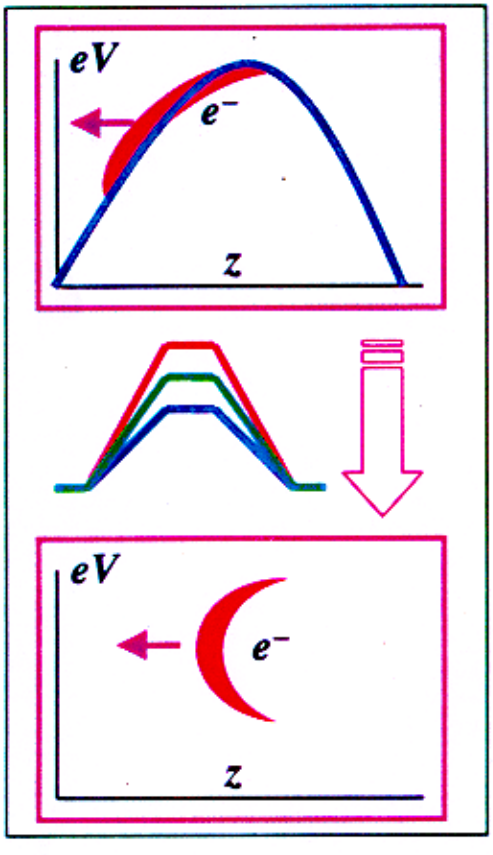
Now add 2nd order term of sinusoidal rf accelerating voltage...

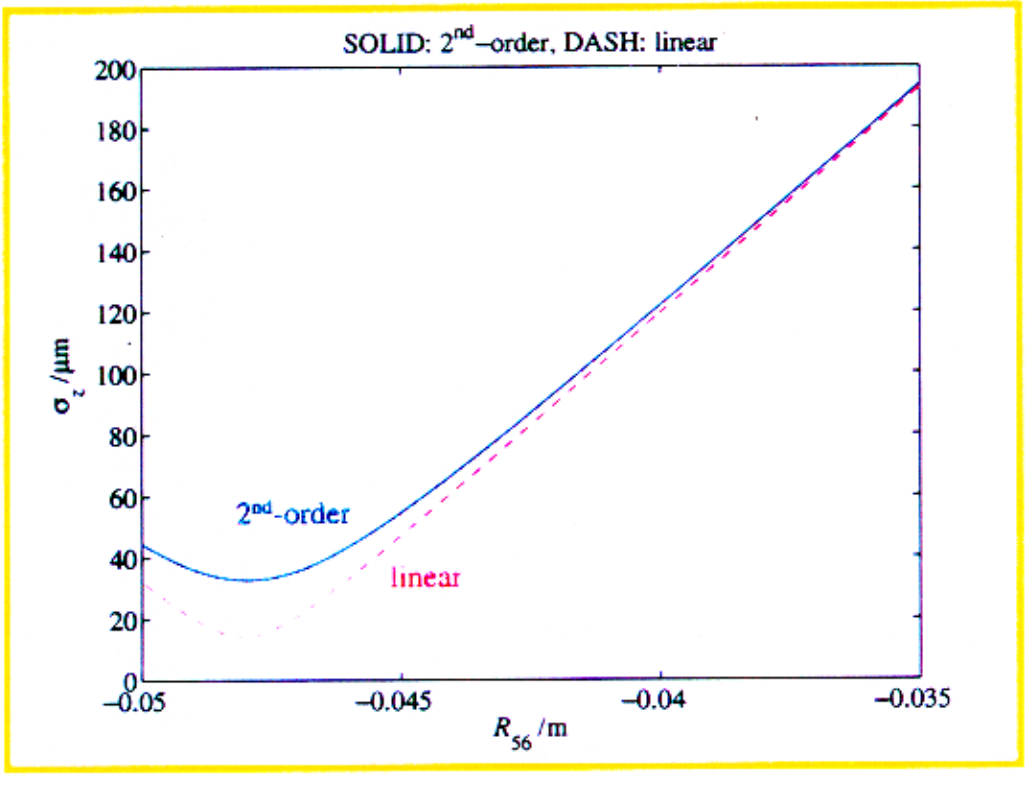
$$\delta \approx \frac{E_0}{E_1}\delta_0 - \frac{2\pi GL\sin\varphi}{\lambda E_1}z_0 - \frac{2\pi^2 GL\cos\varphi}{\lambda^2 E_1}z_0^2$$

For a uniform temporal distribution [$\langle z_0{}^4\rangle = (9/5)\sigma_{z0}{}^4$] and $\langle z_0\delta_0\rangle = 0$ ...

$$\boxed{\sigma_z^2 \approx \left(\frac{E_0}{E_1}\right)^2 R_{56}^2\sigma_{\delta_0}^2 + \left[\left(1-\frac{2\pi R_{56}GL\sin\varphi}{\lambda E_1}\right)^2 + \frac{16\pi^4 R_{56}^2 G^2 L^2}{5\lambda^4 E_1^2}\left(\frac{2rGL\sin^2\varphi}{E_1} - \cos\varphi\right)\sigma_{z_0}^2\right]\sigma_{z_0}^2}$$

For $r < 0$ (chicane: $r = -3/2$) and for an accelerating phase, the RF curvature and T_{566} always add together to limit minimum bunch length...

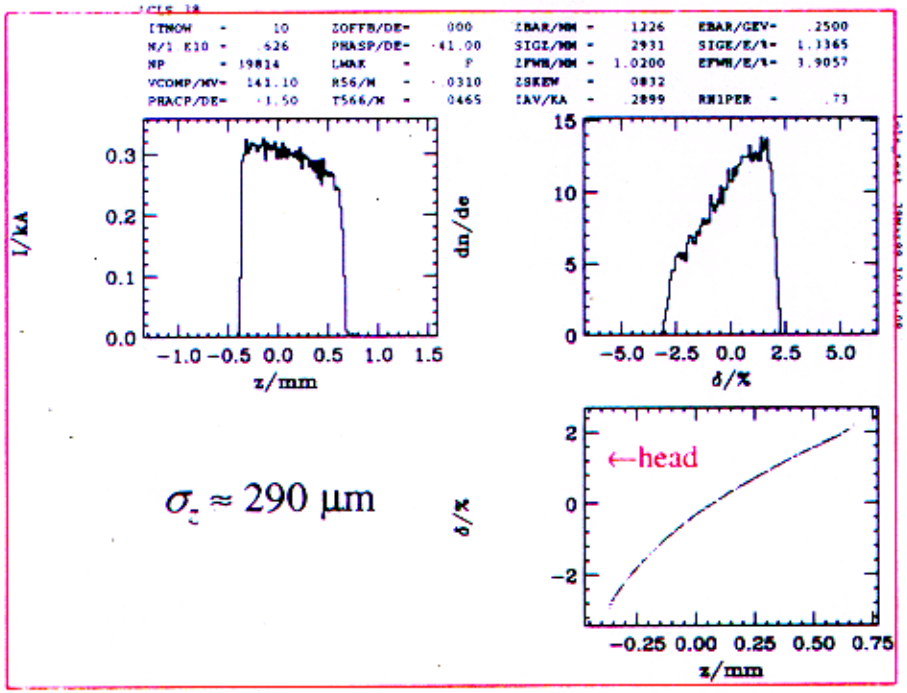




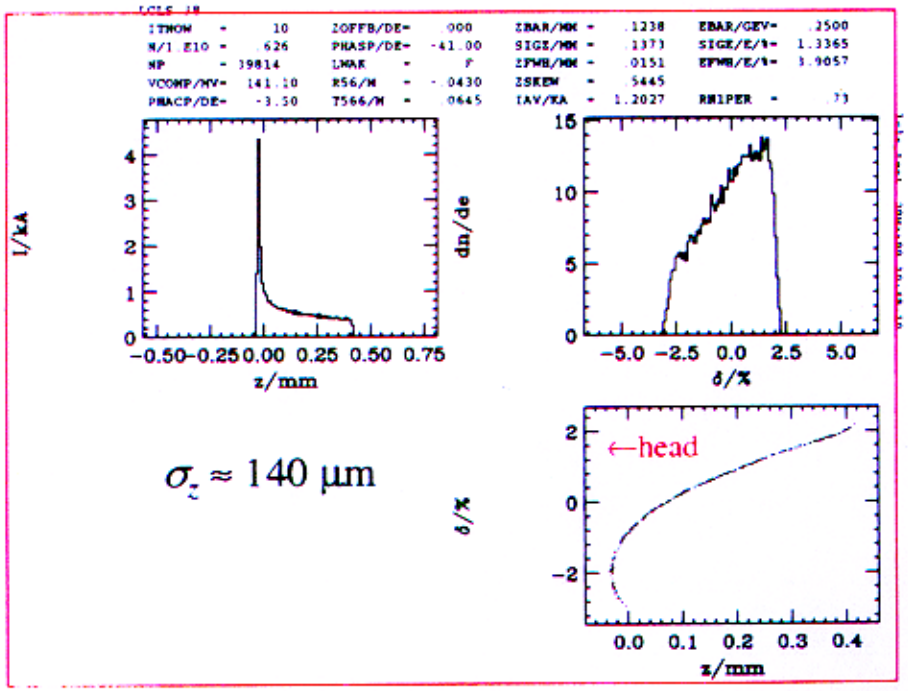
Decelerating phase can be used to compensate T_{566}, but not practical in low energy compressors (used in NLC and possibly TESLA Collider)...

$$\varphi = -\cos^{-1}\left\{\frac{\sqrt{E_0^2 + 8rG^2L^2(1+2r)} - E_0}{2GL(1+2r)}\right\}$$

Example of 2nd Order Effects

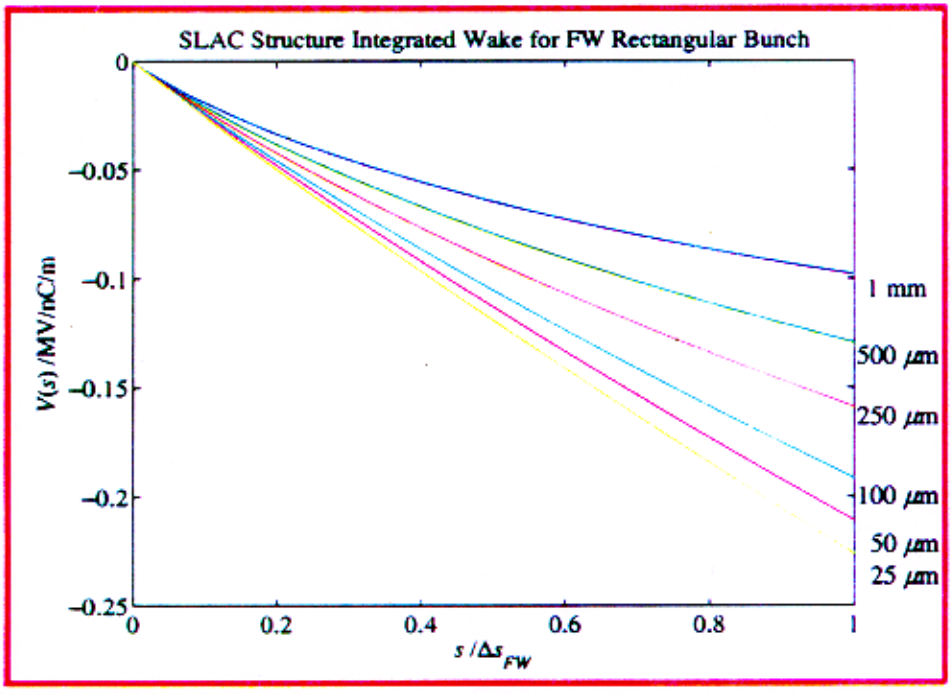


R_{56} tuned properly to setup wakefield cancellation



R_{56} increased too far - bunch folds over itself

Longitudinal Geometric Wakefield



Longitudinal point-wake:

$$W(s) \approx \frac{Z_0 c}{\pi a} e^{-\sqrt{s/s_0}}$$

SLAC S-Band:

$s_0 \approx 1.32$ mm

$a \approx 11.6$ mm

$s < \sim 6$ mm

Induced voltage along bunch:

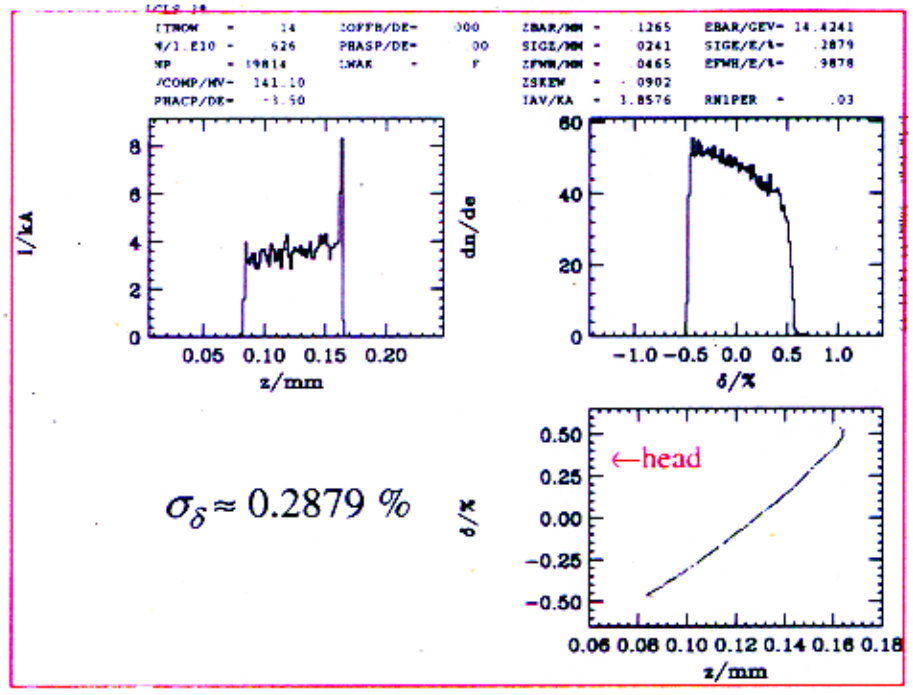
$$V(s) = -NeL \int_{-\infty}^{s} W(s-s') f(s') ds'$$

For a <u>uniform</u> temporal distribution ($\Delta z = 2\sqrt{3}\sigma_z$)...

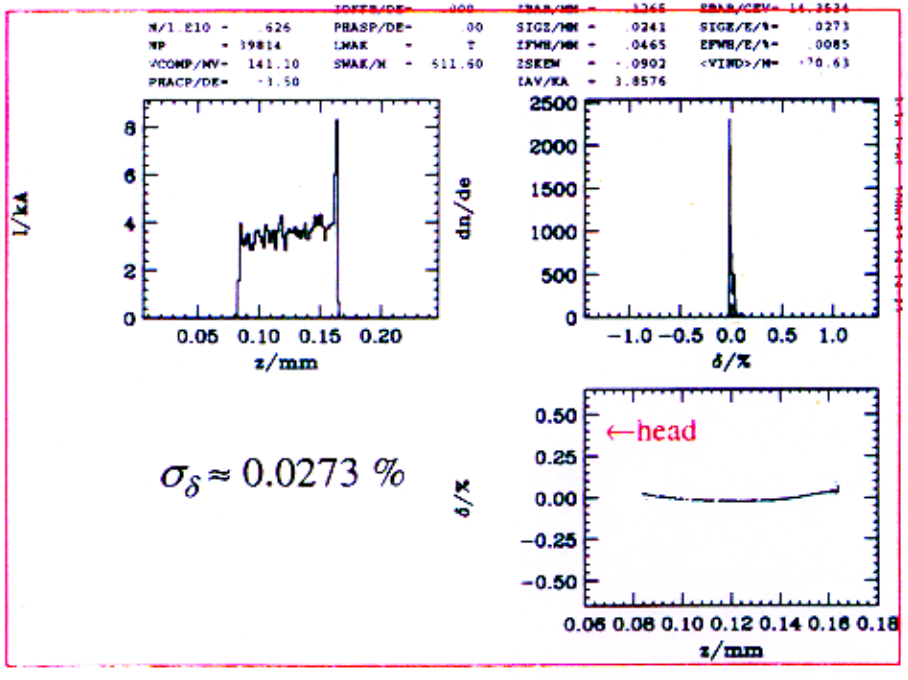
$$k_i \equiv \frac{\partial \delta_i}{\partial s_i} \approx \underbrace{-\frac{2\pi}{\lambda}\left[1 - \frac{E_{i-1}}{E_i}\right]\tan\varphi_i}_{\text{RF slope (+ for chicane)}} \underbrace{- \frac{2Ne^2 c Z_0 s_0 L_i}{\pi a^2 \Delta z^2 E_i}\left[1 - \left(1 + \sqrt{\Delta z / s_0}\right) e^{-\sqrt{\Delta z / s_0}}\right]}_{\text{Wakefield induced slope } (-)}$$

LCLS Example of Wakefield Effect

$L \approx 600$ m, $N \approx 6.25\times10^9$, $\Delta z \approx 80\ \mu$m

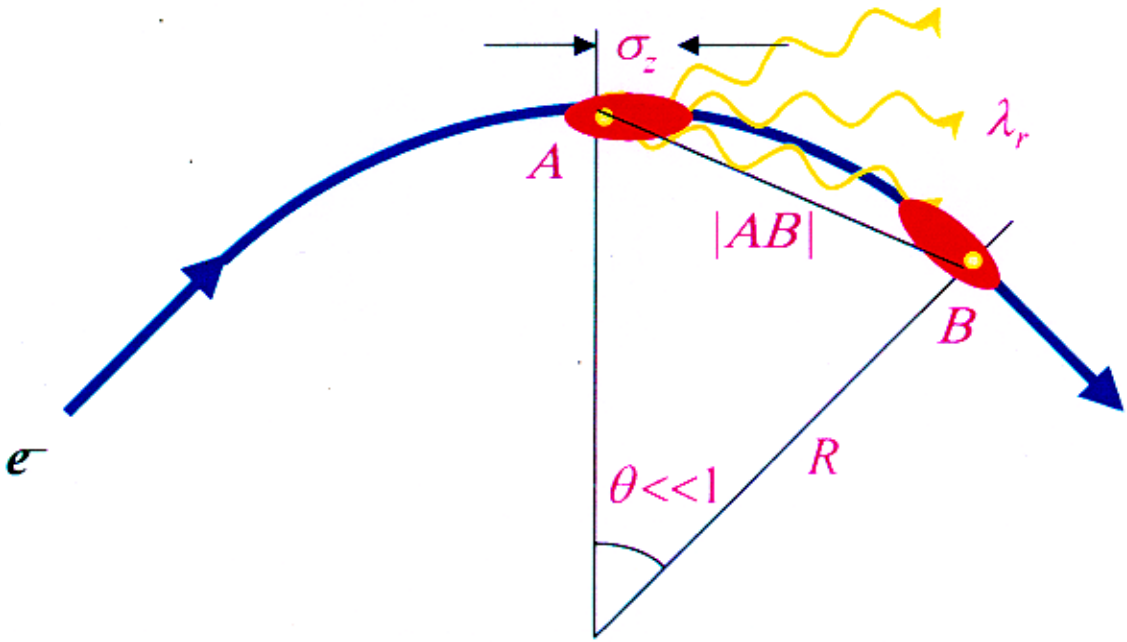


End of linac @ 15 GeV with **wakefield 'OFF'**



End of linac @ 15 GeV with **wakefield 'ON'**

Coherent Synchrotron Radiation (CSR)



Coherent radiation for: $\lambda_r >> \sigma_z$

...from Derbenev, et. al.

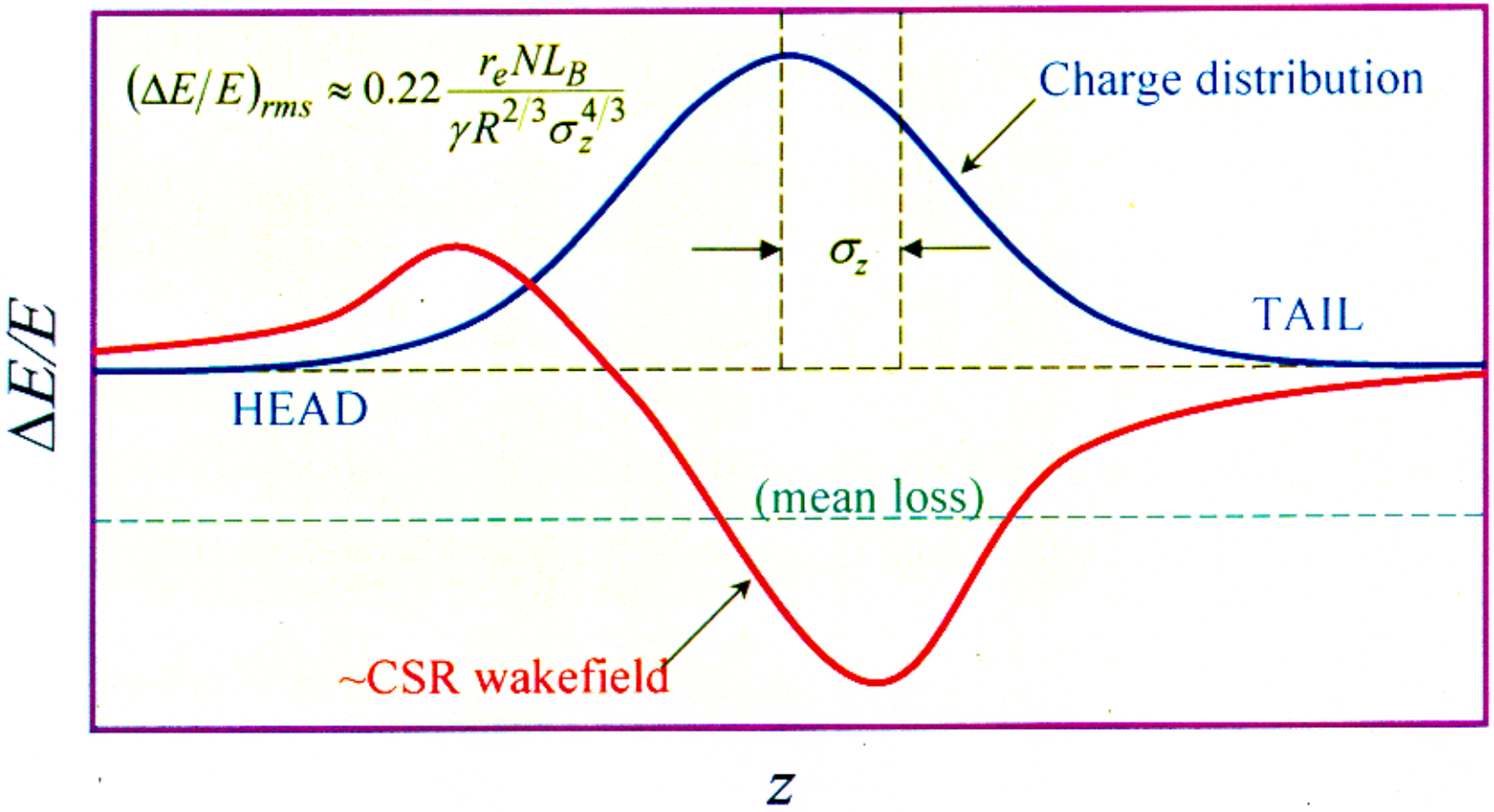
Free space radiation from bunch tail at point ***A*** overtakes bunch head, a distance s ahead of the source, at the point ***B*** which satisfies...

$$s = \mathrm{arc}(\boldsymbol{AB}) - |\boldsymbol{AB}| = R\theta - 2R\sin(\theta/2) \approx R\theta^3/24$$

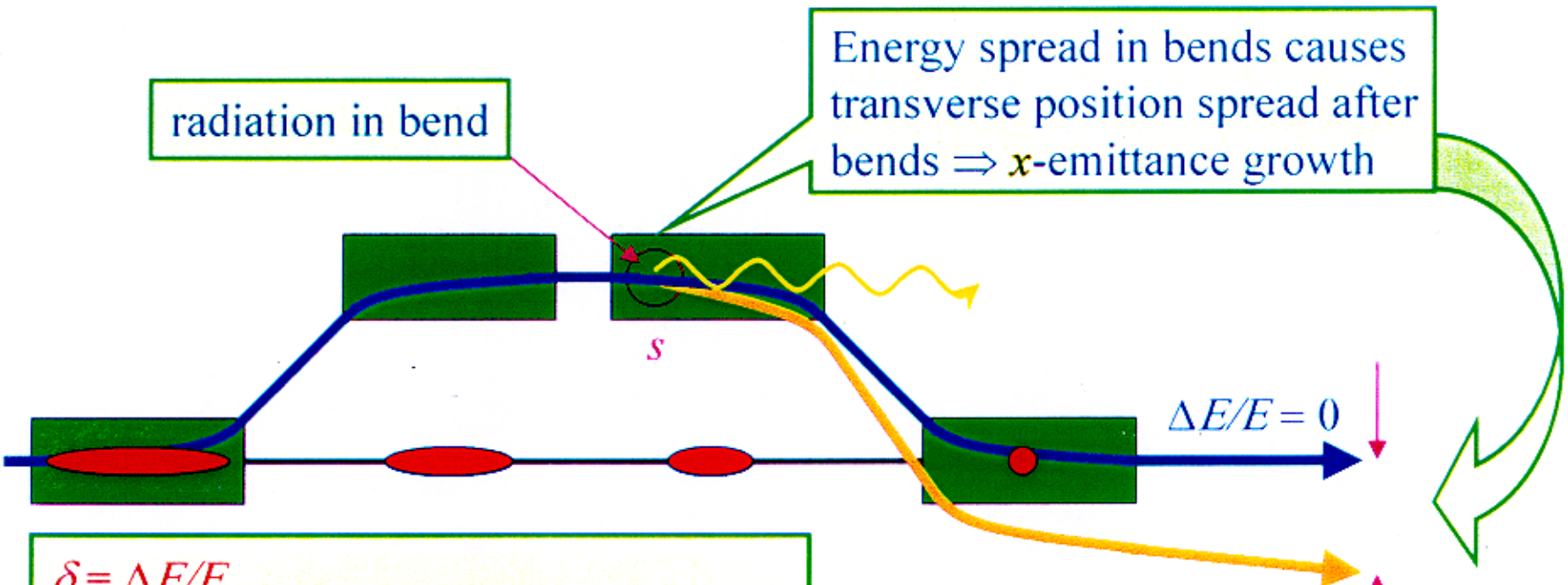
and for $s = \sigma_z$ (rms bunch length) the overtaking distance is...

$$L_0 \equiv |\boldsymbol{AB}| \approx (24\sigma_z R^2)^{1/3}, \quad (\text{HERA: } L_0 > 100 \text{ m}, \ \text{LCLS: } L_0 \sim 1 \text{ m})$$

CSR Effects → Bunch Energy Gradient



CSR Effects - Transverse Emittance Growth



$\delta \equiv \Delta E/E$...

$R_{16}(s) \equiv \partial x/\partial\delta(s), \quad R_{26}(s) \equiv \partial x'/\partial\delta(s)$

$\langle \Delta x^2 \rangle^{1/2} = \int ds R_{16}(s)\, d\sigma_\delta/ds$

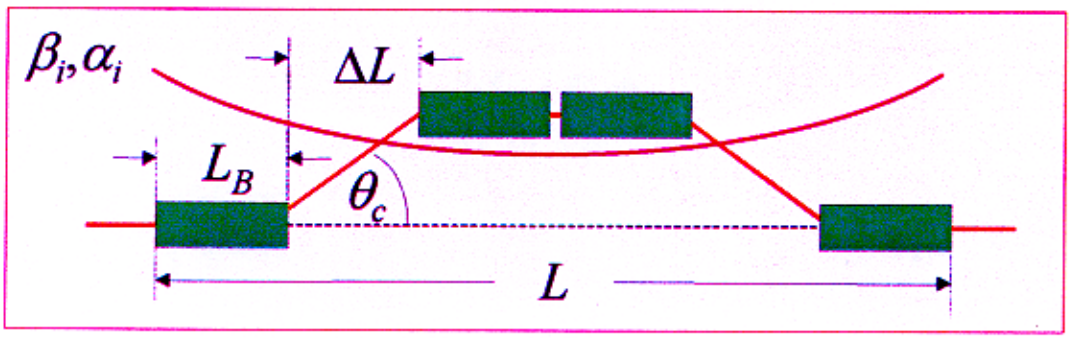
$\langle \Delta x'^2 \rangle^{1/2} = \int ds R_{26}(s)\, d\sigma_\delta/ds$

$\Delta x = R_{16}(s)\Delta E/E$

$\Delta x' = R_{26}(s)\Delta E/E$

$$\varepsilon^2 = \varepsilon_0^2 + \varepsilon_0\{\langle \Delta x^2 \rangle + [\alpha \langle \Delta x^2 \rangle^{1/2} + \beta \langle \Delta x'^2 \rangle^{1/2}]^2\}/\beta$$

CSR Emittance Growth for Chicane



CSR in last bend of chicane (shortest and ~constant σ_z) using steady-state fields...

$$\frac{\varepsilon}{\varepsilon_0} \approx \sqrt{1 + \frac{(0.22)^2}{36} \frac{r_e^2 N^2}{\varepsilon_N \beta_i \gamma} \left(\frac{\theta^5 L_B}{\sigma_z^4} \right)^{2/3} \left[L_B^2 \left(1 + \alpha_i^2\right) + 9\beta_i^2 - 6\alpha_i \beta_i L_B \right]}$$

For small $\Delta\varepsilon/\varepsilon_0$, symmetric beta-functions, and same bend, L_B, and drift, ΔL, lengths...

$$L_B = \Delta L = L/6, \quad \beta_i = L, \quad \alpha_i = +1, \quad b \cong 1.3 \times 10^{-32}\,\mathrm{m}^2$$

$$\boxed{\frac{\Delta\varepsilon}{\varepsilon_0} \approx \frac{bN^2}{\varepsilon_N \gamma} \left(\frac{\theta^{10} L^5}{\sigma_z^8} \right)^{1/3} << 1}$$

R_{56} of same chicane, for $\theta << 1$, given by...

$$R_{56} \approx -2\theta^2\left(\Delta L + \frac{2}{3}L_B\right) = -\frac{5}{9}\theta^2 L$$

Substitute $\theta^2 L$ from above into previous expression for $\Delta\varepsilon/\varepsilon_0$ ...

$$R_{56} \approx -\frac{5}{9}\left(\frac{\gamma\Delta\varepsilon_N}{bN^2}\right)^{3/5}\sigma_z^{8/5}$$

Above is upper limit on chicane $|R_{56}|$ such that CSR normalized emittance increase is $< \Delta\varepsilon_N$.

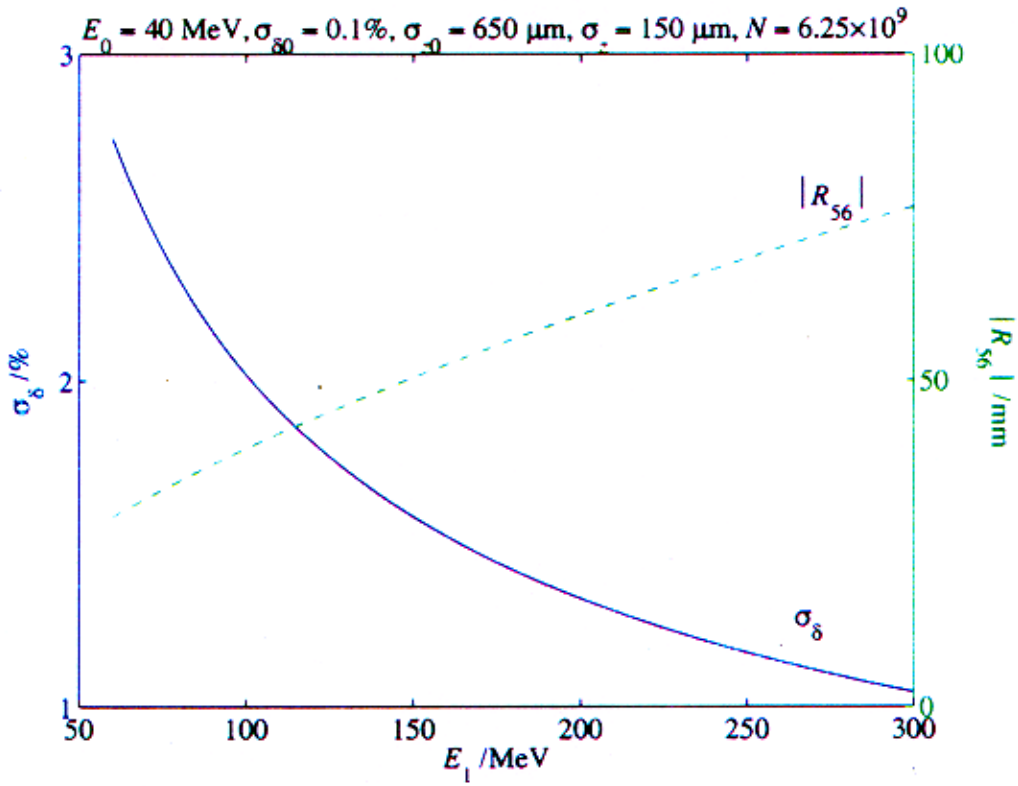
Bunch length and energy spread after single stage compressor...

$$\sigma_z = \sqrt{(1+kR_{56})^2\sigma_{z_0}^2 + R_{56}^2\sigma_{\delta_0}^2\gamma_0^2/\gamma_1^2}, \qquad \sigma_\delta = \sqrt{k^2\sigma_{z_0}^2 + \sigma_{\delta_0}^2\gamma_0^2/\gamma_1^2} \approx |k|\sigma_{z_0}$$

and solve for the energy spread in terms of the tolerable $\Delta\varepsilon_N$ ...

$$\boxed{\sigma_\delta \approx \sqrt{\frac{81}{25}\left(\frac{bN^2}{\gamma_1\Delta\varepsilon_N\sigma_z}\right)^{6/5} - \sigma_{\delta_0}^2\gamma_0^2/\gamma_1^2} + \frac{9}{5}\left(\frac{bN^2}{\gamma_1\Delta\varepsilon_N}\right)^{3/5}\frac{\sigma_{z_0}}{\sigma_z^{8/5}}}$$

APS example of CSR lower limit on chicane energy



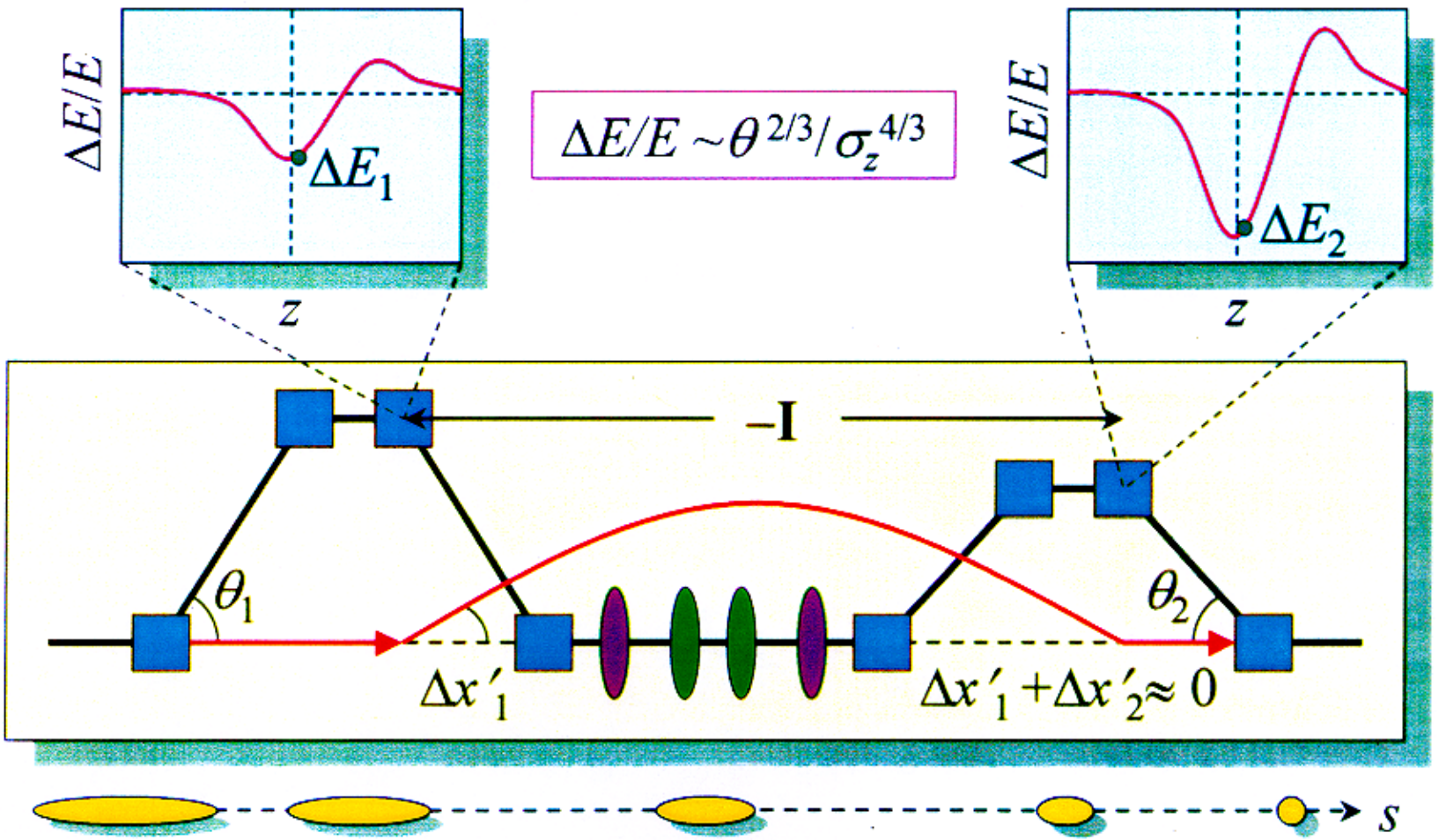
For a fixed CSR emittance increase ($\Delta\varepsilon_N = 0.5\ \mu$m), a chicane located at higher energy reduces the necessary correlated energy spread required for compression.

$\sigma_\delta < 2\% \Rightarrow$
$E_1 > \sim 100$ MeV

$$\sigma_\delta \approx \sqrt{\frac{81}{25}\left(\frac{bN^2}{\gamma_1 \Delta\varepsilon_N \sigma_z}\right)^{6/5} - \sigma_{\delta_0}^2 \gamma_0^2/\gamma_1^2 + \frac{9}{5}\left(\frac{bN^2}{\gamma_1 \Delta\varepsilon_N}\right)^{3/5} \frac{\sigma_{z_0}}{\sigma_z^{8/5}}}$$

$b \cong 1.3\times10^{-32}\,\mathrm{m}^2$
$\Delta\varepsilon_N << \gamma\varepsilon_0$

CSR Emittance Growth Reduction and Cancellation Using Balanced Double Chicane



ISR Emittance Growth for Chicane

Incoherent synchrotron radiation (ISR) increases at high energies and **dilutes the 'slice' emittance**...

$$\frac{\Delta\varepsilon}{\varepsilon_0} \approx aE^6 \frac{|\theta^5|}{\varepsilon_N L_B^2}\left[\Delta L + L_B + \frac{\hat{\beta}+\check{\beta}}{3}\right], \qquad a \cong 8\times10^{-8}\,\mathrm{m}^2\cdot\mathrm{GeV}^{-6}$$

For, symmetric beta-functions, the effect is minimum when...

$$\beta_i = L, \quad \alpha_i = +1, \quad L_B \approx \Delta L \approx L/6$$

$$\frac{\Delta\varepsilon}{\varepsilon_0} \approx 30aE^6 \frac{|\theta^5|}{\varepsilon_N L}$$

And from previous slides, substituting R_{56} for θ...

$$\boxed{L \approx \left[\frac{130aE^6|R_{56}|^{5/2}}{\Delta\varepsilon_N}\right]^{2/7}}$$

Total chicane length, L, has lower limit set by tolerable $\Delta\varepsilon_N$.

LCLS BC2 at $E = 6.00$ GeV and $\Delta\varepsilon/\varepsilon_0 = 1\%$ requires $L \geq 14.3$ m
LCLS BC1 at $E = 0.28$ GeV and $\Delta\varepsilon/\varepsilon_0 = 1\%$ requires $L \geq 0.06$ m

Low Charge LCLS Options

Examine compression modifications of low charge proposal for LCLS (Pellegrini, Ding, Rosenzweig)...

Injector emittance and bunch length scaling with charge, Q ...

$$\gamma\varepsilon \approx (1.45\ \mu\text{m}) \cdot \sqrt{0.38 \cdot Q[\text{nC}]^{4/3} + 0.095 \cdot Q[\text{nC}]^{8/3}}$$

$$\sigma_z \approx (0.63\,\text{mm}) \cdot Q[\text{nC}]^{1/3}$$

$$\gamma\varepsilon \Rightarrow 1\ \mu\text{m}\ (1\ \text{nC}) \rightarrow 0.2\ \mu\text{m}\ (0.1\ \text{nC})$$

$$\sigma_z \Rightarrow 700\ \mu\text{m}\ (1\ \text{nC}) \rightarrow 325\ \mu\text{m}\ (0.1\ \text{nC})$$

- **Same final bunch length at 1.0 nC and 0.1 nC**
- No wakefield left to cancel correlated energy spread in undulator
- Run L2 phase nearer crest to minimize correlated energy spread
- Shorter bunch of injector reduces chicane requirements
- Other wakefields (resistive, transverse geometric, etc.) ~eliminated
- Shorter bunch also reduces energy spread in linacs which ~restores quadrupole alignment tolerances (w.r.t. smaller emittance)

Preliminary study shows that 0.1 nC LCLS operation can be handled without *design* modifications to compressor systems. In fact, most challenges are significantly eased.

Operational modifications...

parameter	symbol	$Q = 1$ nC	$Q = 0.1$ nC	units
Injector bunch length	σ_z	710	330	μm
Injector emittance (rms)	$\gamma\varepsilon$	1.0	0.2	μm
R_{56} of 1st chicane	$R_{56\text{-}1}$	- 31	- 35	mm
RF phase of linac-2	φ_2	- 35	- 22	deg
Bunch length in linac-2	σ_{z2}	290	100	μm
Energy spread in linac-2	$\langle\sigma_{\delta 2}\rangle$	1	0.3	%
R_{56} of 2nd chicanes	$R_{56\text{-}2}$	- 29	- 34	mm
Final bunch length	σ_{zf}	24	24	μm
Final energy spread	$\sigma_{\delta f}$	0.03	0.04	%

The 1-nC parameters are from a recent design (*i.e.* not from the *LCLS Design Study Report*). All other parameters are ~unchanged.

- CSR emittance growth in BC1 is similar...

$$\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)_{CSR} \approx \frac{bN^2}{\varepsilon_N \gamma}\left(\frac{\theta^{10}L^5}{\sigma_z^8}\right)^{1/3} \approx 2\% \cdot \left(\frac{1\,\mu m}{0.2\,\mu m}\right)\cdot\left(\frac{0.1\,nC}{1\,nC}\right)^2 \cdot \left(\frac{290\,\mu m}{100\,\mu m}\right)^{8/3} \approx 2\%$$

- CSR emittance growth in BC2 is maybe reduced...

$$\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)_{CSR} \approx 3\% \cdot \left(\frac{1\,\mu m}{0.2\,\mu m}\right)\cdot\left(\frac{0.1\,nC}{1\,nC}\right)^2 \approx 0.2\%$$

- ISR emittance growth in BC2 is similar (only due to new, lower energy)...

$$\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)_{ISR} \approx 130 a E^6 \frac{\left|R_{56}\right|^{5/2}}{\varepsilon_N L^{7/2}} \approx 1\% \cdot \left(\frac{4.5\,GeV}{6\,GeV}\right)^6 \cdot \left(\frac{1\,\mu m}{0.2\,\mu m}\right) \approx 0.9\%$$

- Quadrupole alignment tolerances in Linac-2 are slightly eased...

$$|\Delta x| \sim \frac{\sqrt{\varepsilon}}{\sigma_\delta} \approx 50\,\mu m \cdot \sqrt{\frac{0.2\,\mu m}{1\,\mu m}} \cdot \left(\frac{1\%}{0.3\%}\right) \approx 70\,\mu m$$

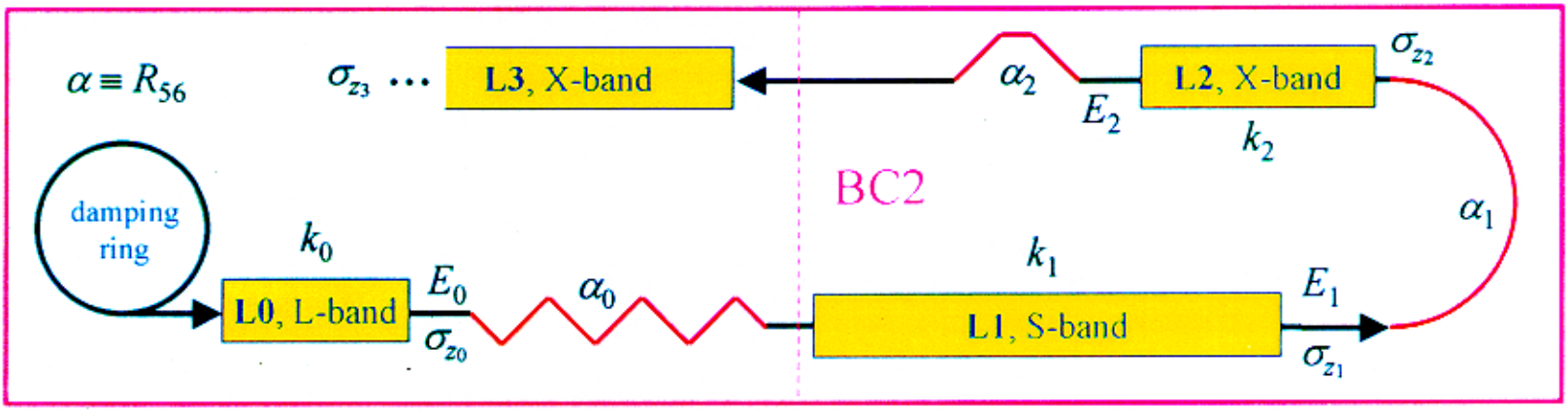
- Transverse wakefield generated emittance growth in linac-2 is reduced...

$$\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)_{\perp} \sim \frac{N^2\sigma_z}{\varepsilon} \approx 15\% \cdot \left(\frac{1\,\mu m}{0.2\,\mu m}\right)\cdot\left(\frac{0.1\,nC}{1\,nC}\right)^2\left(\frac{100\,\mu m}{300\,\mu m}\right) < 1\%$$

- T_{566} non-linearity of BC1 reduced to near insignificance (short bunch in Linac-1)
- Charge jitter tolerance reduced significantly (??? Timing jitter ???)
- Undulator wakefields reduced by a factor of 10.

NLC 2-stage Compressor Optimization

Two-stage compression required to achieve final bunch length with reasonable jitter tolerances and reduced space charge effects...



QUESTIONS: 1) Short-arc/long-L2, or long-arc/short-L2 ?
2) Best arc energy ?

$$\mathbf{R}(\mathrm{BC2}) = \begin{bmatrix} (1+\alpha_1 k_1)(1+\alpha_2 k_2) + \alpha_2 k_1 E_1/E_2 & \alpha_1(1+\alpha_2 k_2)E_0/E_1 + \alpha_2 E_0 \;\; E_2 \\ k_2(1+\alpha_1 k_1) + k_1 E_1/E_2 & \alpha_1 k_2 E_0/E_1 + E_0 \;\; E_2 \end{bmatrix}$$

Jitter requirements (L0 E-jitter $\neq$ L3 φ-jitter): $R_{12} = 0 \Rightarrow$ $\alpha_1 = -\dfrac{E_1}{E_2}\dfrac{\alpha_2}{1+\alpha_2 k_2}$

Compression factor, M, requirements: $R_{11} = \pm 1/M \Rightarrow$ $1+\alpha_2 k_2 = \pm 1/M \equiv \pm \dfrac{\sigma_{z_3}}{\sigma_{z_1}}$

With each k_i defined as...

$$k_i = \frac{2\pi}{\lambda_i E_i} G_i L_i \sin\varphi_i$$

With φ_2 chosen for T_{566} compensation (as described earlier) the R_{56} (α_1) of the arc is then explicitly given by...

$$\alpha_1 = \frac{E_1(M-1)\lambda_2}{2\pi G_2 L_2 |\sin\varphi_2|}$$

The R_{56} of the arc is related to the arc length, L_a, the number of FODO-cells, N_c, and the x-phase advance per cell, μ, as ...

$$\alpha_1 \approx \frac{\pi^2 L_a}{4N_c^2 \sin^2(\mu\ 2)}$$

And the ISR emittance growth of the arc is approximately ...

$$\Delta\gamma\varepsilon \approx \frac{aE^6\pi^5 \cos(\mu\ 2)}{8N_c^3 L_a p^2 \sin^3(\mu\ 2)}$$

with p as an arc dipole packing factor (~0.7).

The energy at the BC2 chicane is ($E_2 < E_1$ for decelerating phase)...

$$E_2 = E_1 + G_2 L_2 \cos\varphi_2$$

and the R_{56} of the chicane is related to that of the arc by...

$$\alpha_2 = -\frac{\alpha_1 E_2}{M E_1}$$

which sets the total chicane length at...

$$L_c \approx \left[\frac{130 a E^6 |\alpha_2|^{5/2}}{\Delta\varepsilon_x} \right]^{2/7}$$

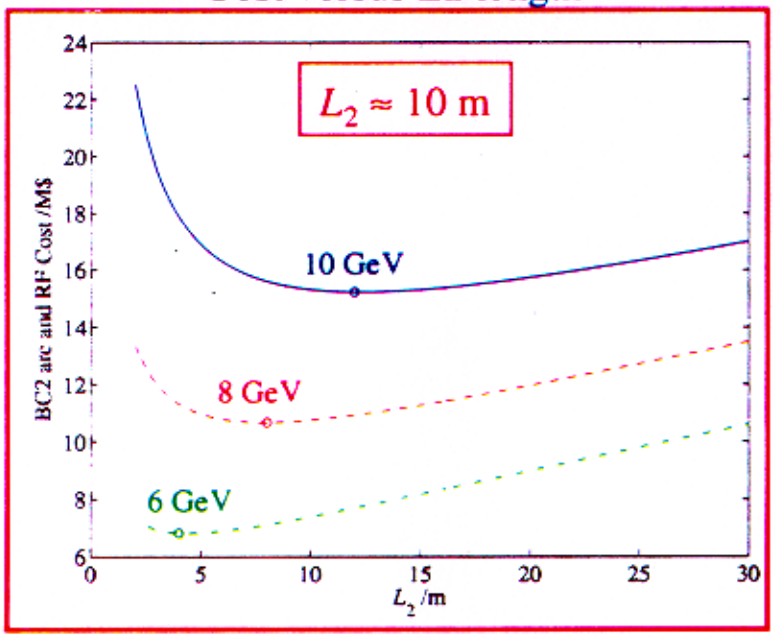
$$\boxed{\sigma_{z0} \approx 5 \text{ mm} \Rightarrow \sigma_{z1} \approx 0.5 \text{ mm} \Rightarrow \sigma_{z2} \approx 90\ \mu\text{m}}$$

The significant free parameters are the arc-energy, E_1, and the length of the post-arc X-band RF section, L_2.

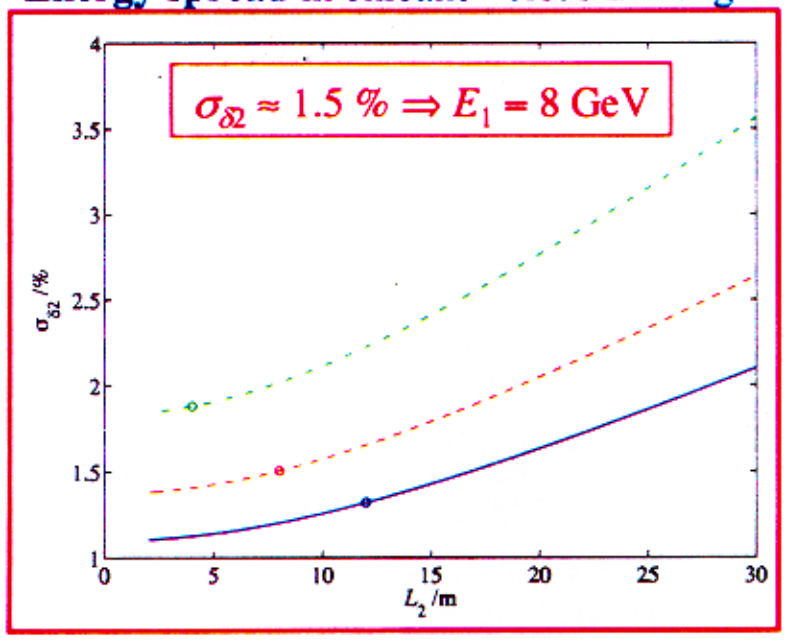
These are chosen by a cost optimization including 1) cost per meter of X-band RF, 2) cost per meter of arc, 3) cost per arc FODO-cell, and 4) cost per meter of chicane ...

$$\text{Cost} \approx A \cdot L_2 + B \cdot L_a + C \cdot N_c + D \cdot L_c$$

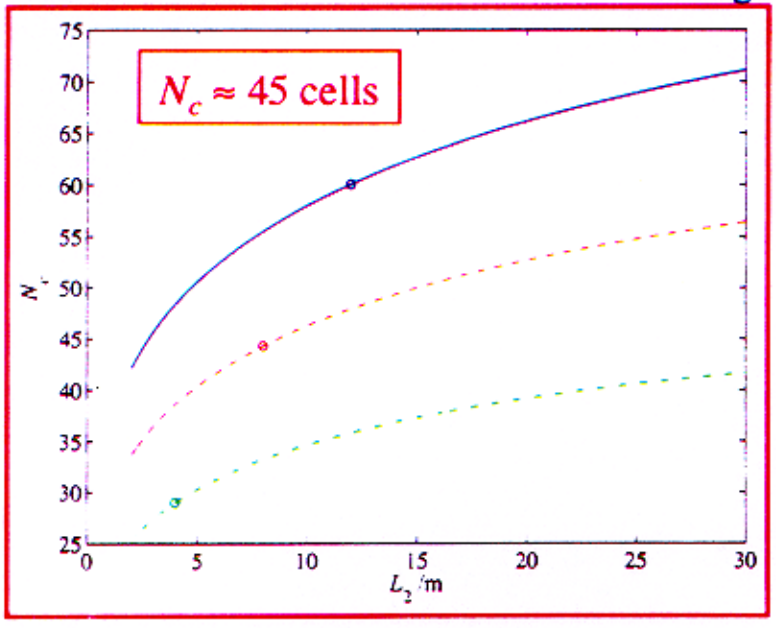
Cost versus L2 length



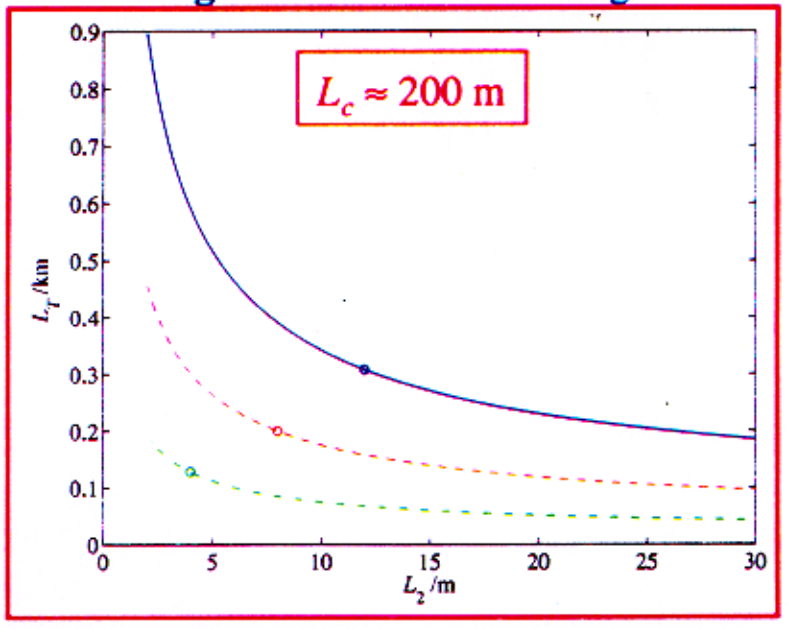
Energy spread in chicane versus L2 length



Number of arc FODO-cells versus L2 length



Length of arc versus L2 length



Summary

- Many design possibilities exist
- 2nd order compensation possible (high energy)
- Jitter sensitivity minimization can force multiple stages
- Wakefield must be considered, and can be used to remove correlated energy spread
- Designs exist to reduce synchrotron radiation effects
- Low charge looks favorable for accelerator and FEL
- Some scaling laws available for cost and performance optimization