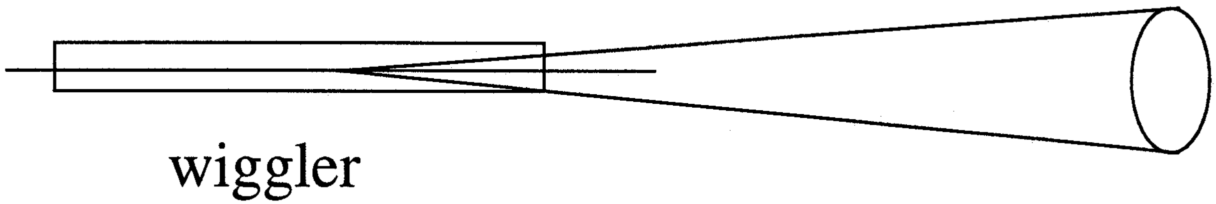
Max Zolotorev

Radiation Tutorial

- Diffraction
- Formation Length
- Volume of coherence
- Parameter of degeneracy
- Small signal small gain
- Exponential gain
- Saturation
- Gamma distribution
- Fluctuations in intensity
- Spectral fluctuations

Transverse Coherence and Formation Length



In far field region observer see radiation with θ_R

$$\theta_R = \frac{\lambda}{d_{tr}} \Longrightarrow$$ Diffraction limited angle

$$d_{tr} \Longrightarrow$$ Transverse coherence size

$$L_W \propto L_F \Longrightarrow$$ Formation length of radiation

$$L_F = \frac{d_{tr}}{\theta_R} = \frac{\lambda}{\theta_R^2} \Longrightarrow \theta_R \propto \sqrt{\frac{\lambda}{L_W}}; \quad d_{tr} \propto \sqrt{\lambda L_W}$$

Source view

$E_s(t,r)$ $\Longrightarrow$ Electric field of spontaneous emission of one electron

$\sum E_s(t-t_i,r-r_i)$ $\Longrightarrow$ Electric field of spontaneous emission of the beam

$$\sum E_s(t-t_i,r-r_i) \Longrightarrow \sum E_s(\omega,\theta)\, e^{i(\omega\tau_i - \vec{k}\vec{r}_i)} \Longrightarrow \sum E_s(\omega,\theta)\, e^{i(\omega\tau_i - k\theta\, r_i)}$$

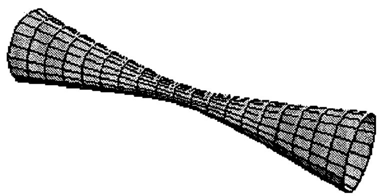
$$k\theta r < 1 \Longrightarrow r < \frac{\lambda\!\!\!^{-}}{\theta} = d_{tr} \Longrightarrow$$ Field from all electrons in transverse dimensions adds in phase (coherently)

$$\frac{\Delta\omega}{\omega} \approx \frac{1}{M_W};\quad l_{long} = \frac{c}{\Delta\omega} \approx \lambda\!\!\!^{-} M_W \Longrightarrow$$ Longitudinal length of coherence

$d_{tr}^2\, l_{long}$ $\Longrightarrow$ Volume of coherence (mode volume)

Gaussian beam approximation

Light optics



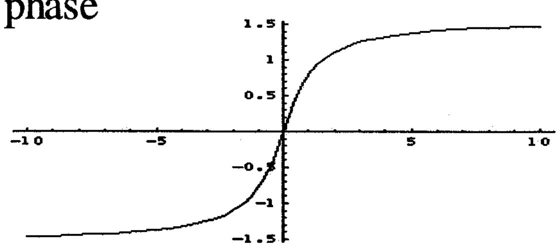
$$\varepsilon_R = \frac{\lambda}{4\pi} = \frac{\lambda\!\!\!^{-}}{2}$$

$$\sigma_{trR}^2 = \varepsilon_R Z_R$$

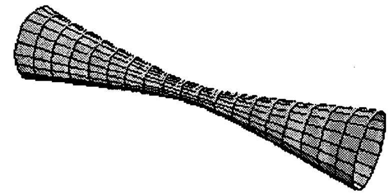
$$\sigma_{\theta R}^2 = \frac{\varepsilon_R}{Z_R}$$

$$\sigma_{trR}^2(z) = \sigma_{trR}^2(0)\left[1 + \frac{z^2}{z_R^2}\right]$$

Guoy phase



Accelerator optics



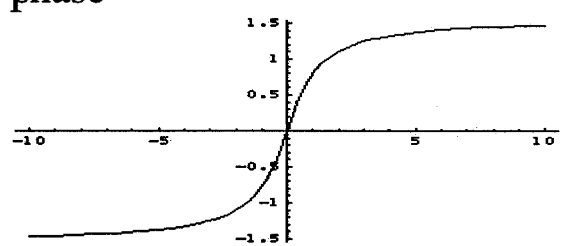
$$\varepsilon_b = ? >>>> \frac{\lambda\!\!\!^{-}_c}{2}$$

$$\sigma_{trb}^2 = \varepsilon_b \beta$$

$$\sigma_{\theta b}^2 = \frac{\varepsilon_b}{\beta}$$

$$\sigma_{trb}^2(z) = \sigma_{trb}^2(0)\left[1 + \frac{z^2}{\beta^2}\right]$$

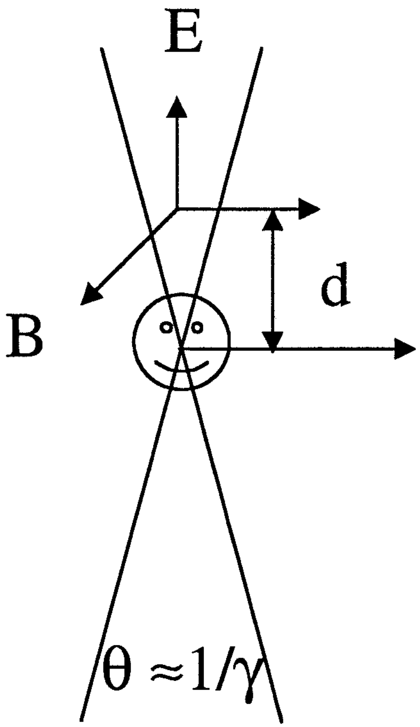
Betatron phase



For best matching of Gaussian beam and wiggler radiation

$$Z_R = L_W/4$$

Intensity (method of equivalent photons)



Plane wave but v≠c

$$\frac{1}{\Delta t} = \frac{c\gamma}{d} = \Delta\omega = \omega$$

V ≈ C

$$d = \lambda\!\!\!^{-}\gamma = \frac{\lambda\!\!\!^{-}}{\theta} = d_{tr}$$

In one mode α of photons $\quad \alpha = \dfrac{e^2}{\hbar c} \approx \dfrac{1}{137}$

If electron are deflected by angle ϕ

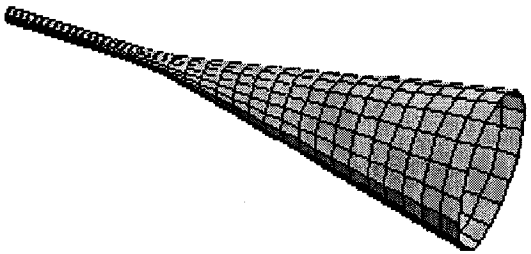
it can radiate all or fraction of his cloud

$$\frac{\phi^2}{\phi^2 + \frac{1}{\gamma^2}} \Rightarrow \frac{K^2}{1+K^2}$$

were K is deflection parameter

Energy radiated per mode $\Longrightarrow \quad 4\alpha\,\hbar\omega \dfrac{K^2}{1+K^2} = 4\dfrac{K^2}{1+K^2}\dfrac{e^2}{\lambda\!\!\!^{-}}$

Small signal and small gain



In far field region we have fields from external source and spontaneous emission of electron $E = E_L + E_S$

$$\text{Energy} = \frac{c}{4\pi}\int |E|^2 d\Omega d\omega = \mathcal{E}_l + \mathcal{E}_s + 2\sqrt{\mathcal{E}_l \mathcal{E}_s}\ \mathrm{Cos}(\varphi_i)$$

$$\Delta\mathcal{E}_e = 2\sqrt{\mathcal{E}_l \mathcal{E}_s}\ \mathrm{Cos}(\varphi_i)$$

$$E_S \Longrightarrow E_S + \frac{\partial E_s(\omega - \omega_e(\mathcal{E}_e))}{\partial\, \mathcal{E}_e}\Delta\, \mathcal{E}_e$$

as a result

$$g(\omega) = 2\pi\, \dot{N}\, \frac{dW_s(\omega - \omega(\mathcal{E}_e))}{d\ \mathcal{E}_e} \qquad \int_{-\infty}^{\infty} W_s(\omega) d\omega = \mathcal{E}_s$$

For wiggler based free electron laser

For zero emittance beam

$$g^{o}(\omega) \approx \frac{K^2}{1+K^2}(2\pi M_W)^2 \frac{I}{\gamma I_A}\frac{d}{dx}\frac{Sin^2(x-\sqrt{2})}{(x-\sqrt{2})^2}$$

$$x = \pi M_W \frac{\omega - \omega_e}{\omega_e}$$

Shift in frequency related to correlation of radiated energy with angle (far field view) or Guoy phase shift (near field view)

$$\frac{dI}{d\Omega} \propto \frac{Sin^2\left[\pi M_W\left(\frac{\Delta\omega}{\omega} + \frac{\theta^2\gamma^2}{1+K^2}\right)\right]}{\left[\pi M_W\left(\frac{\Delta\omega}{\omega} + \frac{\theta^2\gamma^2}{1+K^2}\right)\right]^2}$$

Red shift (Thomson scattering)

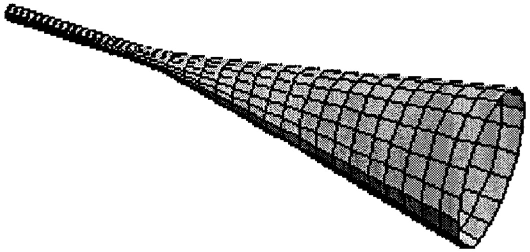
$$\beta_{phase} = 1 + \frac{1}{kZ_R}$$

Guoy phase shift $\Longrightarrow$ Red shift

and for final emittance beam

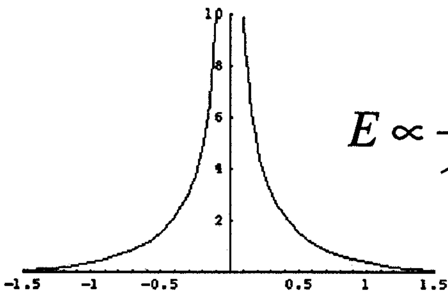
$$g(\omega) \approx g^{o}(\omega)\frac{1}{1+\frac{\varepsilon_b}{2\varepsilon_R}(\frac{\beta}{Z_R}+\frac{Z_R}{\beta})}$$

Small signal Exponential gain



Field look like field in optical fiber amplifier

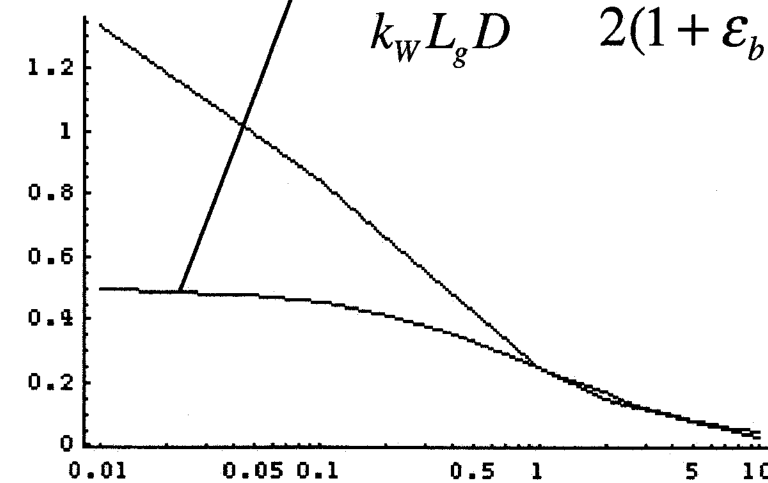
$$\theta_{dif} = \sqrt{\frac{\lambda\!\!\!^{-}}{2L_g}}\,; \quad d_{tr} = \sqrt{\frac{\lambda\!\!\!^{-} L_g}{2}}$$



$$E \propto \frac{1}{\sqrt{r}}\, e^{\frac{z}{L_g} - \frac{r}{\sqrt{\lambda\!\!\!^{-} L_g}}}$$

$$\frac{1}{k_W L_g D}$$

$$\frac{1}{k_W L_g D} = \frac{1}{2(1+\varepsilon_b/\varepsilon_R)}$$



$$\frac{\varepsilon_b}{\varepsilon_R}$$

for final emittance beam

Yong Ho Chin, Kwang-Je Kim, Ming Xie

$$D = \sqrt{\frac{K^2}{1+K^2}\frac{8I}{\gamma I_A}}$$

Radiated power

$P = I^2 Z_0 M_g b^2(z)$

$b^2(z) = b_0^2 \, \text{Exp}\,[2z/L_g]$

$Z_0 = 377\ \Omega$

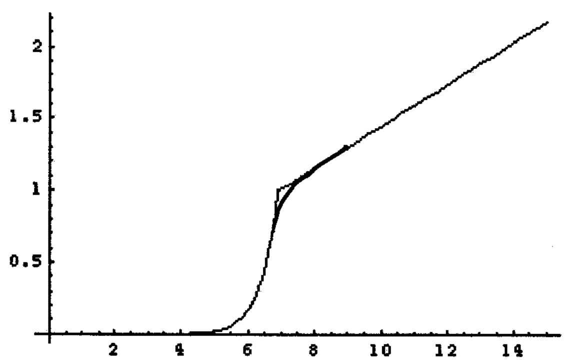
$b_0^2 = 1/N_s$

$$\frac{\Delta\omega}{\omega} = \frac{1}{M_g}$$

$$N_s = \dot{N}\, 2\pi\, \omega\, M_g$$

$b_0^2 \, \text{Exp}\,[2L_{sut}/L_g] = 1$

$2\, L_{sut}/L_g = \text{Log}[N_s]$



After saturation: power grow linearly and shape of the mode changes

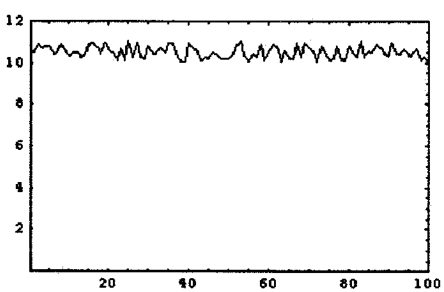
$2\, L_{sut}/L_g$

Spectral distribution

For small signal (before saturation) system work like linear amplifier and spectral width became narrower (Power narrowing)

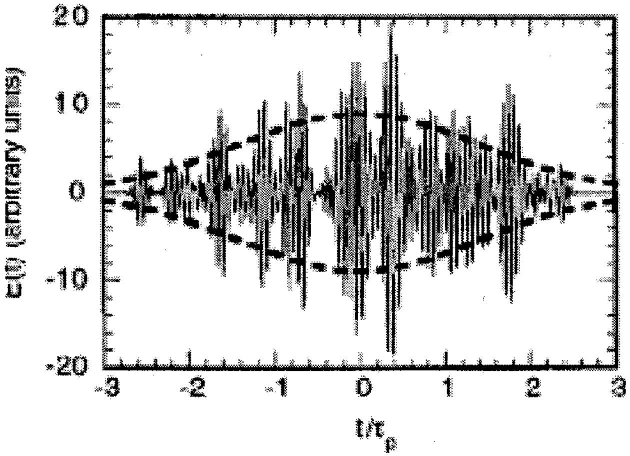
$$\frac{\Delta\omega}{\omega} \approx \frac{1}{M_g} \frac{1}{\sqrt{Log[Gain]}}$$

After saturation spectral width became wider. Why ?



DC current can not radiate. Vacuum look like capacitor Dc current can not go threw. And radiation look like all another useful things)

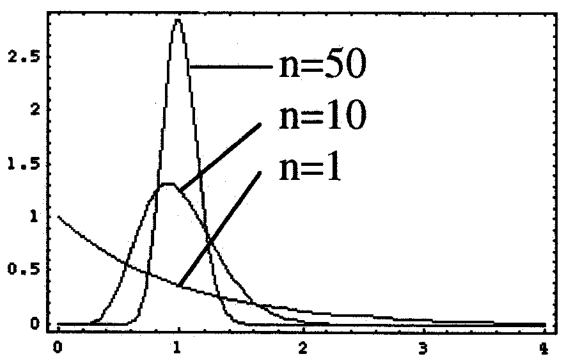
coming from noise



This how look input signal for amplifier. Before saturation, system was linear, and as result of slippage, bunching became superposition of this noise.
After saturation (bunching can not be >1) different pieces of noise start compete with each other and destroyed bunching. As a result is spectral broadening.

Number of spikes is $n \approx \dfrac{c\tau_b}{\lambda M_g \sqrt{Log[gain]}}$ (in linear case)

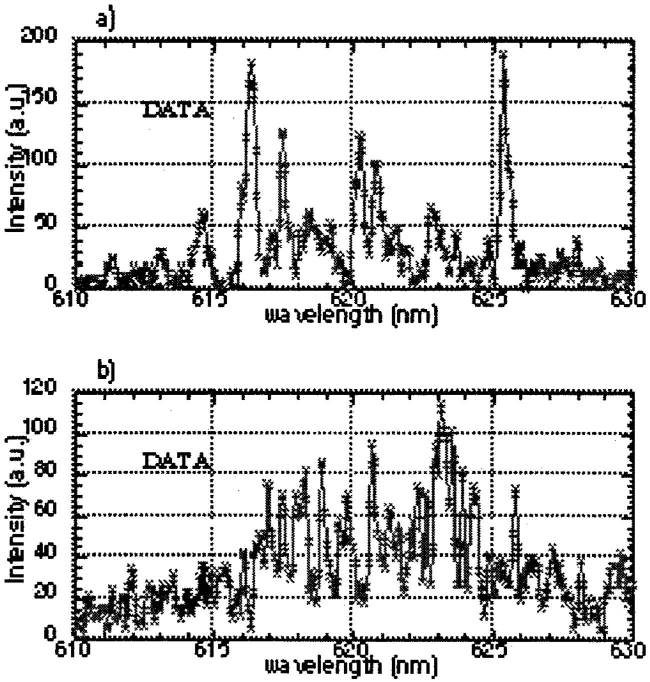
Each spike are independent and fluctuation of normalized intensity will follow of distribution of sum **n** independent Poisson process Gamma[n] distribution.



$$f(x;n) = \frac{x^{n-1} n^{n} e^{-nx}}{\Gamma(n)}$$

$$< x >= 1; \quad Variance = n$$

Spectral fluctuation



Spectral fluctuations: narrow spicks with width $1/\tau_{b.}$ In case of pure resolution of spectrometer or mixing radiation from source large than transverse coherence size or both distribution of normalize intensity of spikes will be Gamma distribution

Palma Catravas et. all

Intensity, gain length (in number of wiggles), statistical property and so on does not depend from wave length of radiation. Real dependencies is emittance and wiggler period .To date best achieved normalize emittance for reasonable charge is 1 π mm mrad, and wiggler period few mm and because we are not smart enough to make good source of electrons and microwiggler we are forced go to really

Very High Energy for X-ray Free Electron Laser.