Parameter Optimization for a 1.5 to 0.15 nm FEL for a Very Low Slice Emittance

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April 7, 1999

- Slice Emittance
- Optimization Algorithm
- FEL Optimization Results
- Comparison with Simulation
- Sensitivities to Parameter Changes

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Analytical 3D FEL Formula

Ming Xie, LBNL, determined universal FEL scaling by fitting numerical solutions of the coupled Maxwell-Vlasov equations that describe the FEL interactions.

$$\eta = 0.35\,\eta_d^{0.57} + 0.55\,\eta_\varepsilon^{1.6} + 3\,\eta_\gamma^{2} + 0.35\,\eta_\varepsilon^{2.9}\eta_\gamma^{2.4} + 51\,\eta_d^{0.95}\eta_\gamma^{3} + 5.4\,\eta_d^{0.7}\eta_\varepsilon^{1.9} + 1140\,\eta_d^{2.2}\,\eta_\varepsilon^{2.9}\eta_\gamma^{3.2}$$

The scaling parameters

$$\eta_d = \frac{L_{1d}}{L_R}, \quad \eta_\varepsilon = \left(\frac{L_{1d}}{\beta_{xy}}\right)\left(\frac{4\pi\varepsilon_n}{\gamma_r\,\lambda_r}\right); \quad \eta_\gamma = 4\pi\left(\frac{L_{1d}}{\lambda_u}\right)\left(\frac{\sigma_E}{E_o}\right)$$

measure the deviation of the beam from the 1-D case.

$$L_{G3D} = (1+\eta)L_{G1D}; \qquad P_{sat} \approx \frac{1.6\,\rho}{(1+\eta)^2}P_{beam};$$

$$L_{sat} \approx \log\left(\frac{P_{sat}}{\rho\,\gamma\,m_e c^2\,\Delta\omega}\right)L_{G3D}$$

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1D FEL Formulae

1D Power Gain Length:

$$L_{G1D} = \frac{\lambda_u}{4\pi\sqrt{3}\,\rho}$$

Pierce Parameter:

$$\rho = \frac{1}{4}\left(\frac{a_w}{4\gamma}\frac{\Omega_p}{\omega_u}F_1\right)^{\frac{2}{3}}$$

Bessel Function Term:

Helical: $F_1 = 1$

Linear: $F_1 = J_0(\xi/2) - J_1(\xi/2)$

$$\xi = \frac{a_w^2}{1 + a_w^2}$$

Undulator Wavenumber:

$$k_u = \frac{2\pi}{\lambda_u} \qquad \omega_u = k_u\, c$$

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1D FEL Formulae

Undulator Parameter:

$$K = \frac{B_u \, c}{k_u \left(m_e \, c^2 / e \right)}$$

Undulator Strength:

Helical: $a_w = K$

Linear: $a_w = K / \sqrt{2}$

β-Function for Natural Focusing:

$$\beta_{nat} = \sqrt{2} \, \frac{\gamma}{K \, k_u}$$

Total Betatron Wavenumber:

$$k_\beta = \sqrt{\left(\frac{1}{\beta_{ext}} \right)^2 + \left(\frac{1}{\beta_{nat}} r_{kxkw} \right)^2}$$

Total β-Function:

$$\beta_{xy} = \frac{1}{k_\beta}$$

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1D FEL Formulae

Raleigh Range:

$$L_R = \frac{\pi \, w_o^{\,2}}{\lambda_r}$$

Optical Waste Size:

$$w_o = \sqrt{2}\sqrt{\sigma_{xy}^{\,2} + 2\,\frac{\lambda_r L_{G1D}}{(4\pi)^2}}$$

Plasma Frequency:

$$\Omega_p = \sqrt{\frac{4\pi \, r_e \, c^2 \, n_e}{\gamma}}$$

Electron Density:

$$n_e = \frac{I_{pk}}{2\pi \, \sigma_{xy}^{\,2} \, e \, c}$$

RMS Electron Beam Size:

$$\sigma_{xy} = \sqrt{\frac{\varepsilon_n}{\gamma}\beta_{xy}}$$

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1D FEL Formulae

Resonant Condition:

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + a_w{}^2\right)$$

Classical Electron Radius:

$$r_e = \frac{e\ c^2}{(m_e\, c^2 / e)} 10^{-7}$$

Frequency Spread:

$$\Delta\omega = 2\frac{c}{\lambda_r}\frac{\Delta\gamma}{\gamma}$$

Parameters and Constants:

λ_r	Resonant Wavelength
γ_r	Resonant Energy
$\Delta\gamma_r = \sigma_E$	Energy Spread
ε_n	Normalized Emittance
λ_u	Undulator Period
B_u	Peak On-Axisis Undulator Field
β_{ext}	External Focusing Strength
I_{pk}	Peak Bunch Current
m_e	Electron Mass
c	Speed of Light
e	Electron Charge

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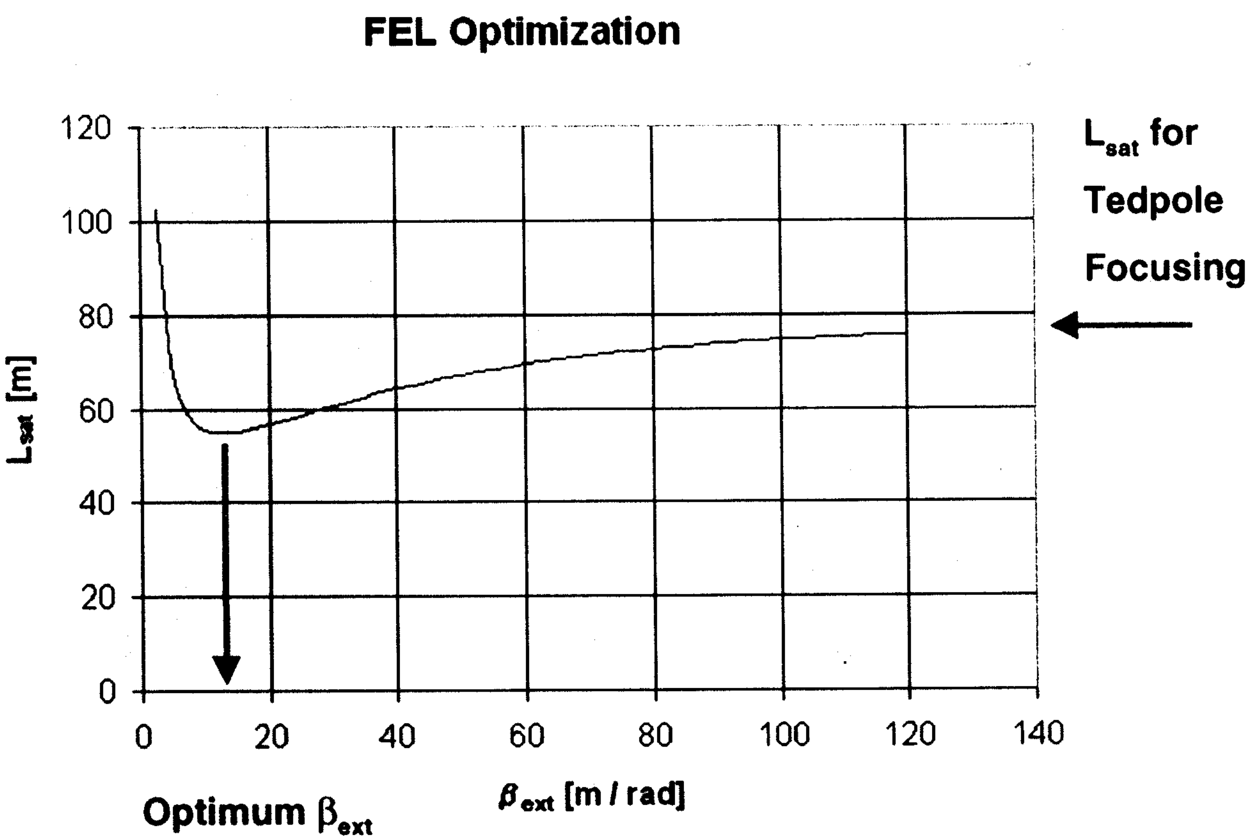
Optimum Focusing



λ_r = 0.15 nm

λ_u = 2.7 cm

I_{pk} = 3400 A

ε_n = 1.5 mm mrad

Helical Undulator

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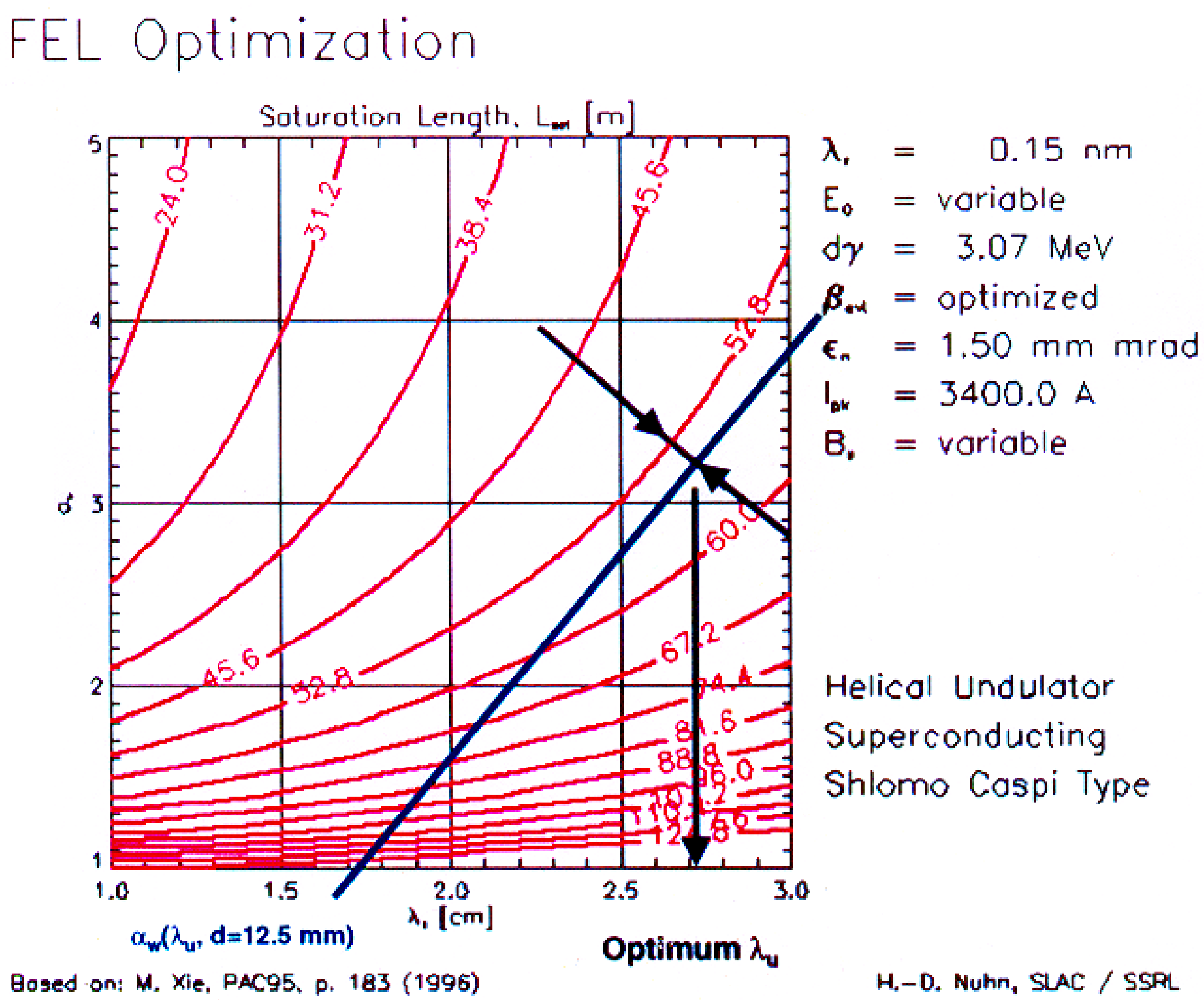
Optimum Period

Optimum period exists for constant diameter or gap.



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FEL Parameter Classification

- **Case Variables**
  - λ_r
  - gap / diameter
  - undulator type
  - $d\gamma$
- **Optimized Variables**
  - λ_u
  - β_{ext}
- **Free Variables**
  - γ_r
  - a_w
  - B_u
- **Plot Variables** (vs. I_{pk}, ε_n)
  - λ_r, γ_r, a_w, B_u, L_{sat}, P_{sat}

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FEL Optimization Cases

- Wavelengths Range

$\lambda_r = 0.1, 0.15, 1.5, 15$ nm

- Fixed Energy Spread

$d\gamma = 6$

Undulator Type	Gap / Diameter	Quad. Foc. , β_{ext}
	[mm]	
linear	6	optimized
linear	6	none
linear	7	optimized
linear	7	none
helical	10	optimized
helical	10	none
helical	12.5	optimized
helical	12.5	none

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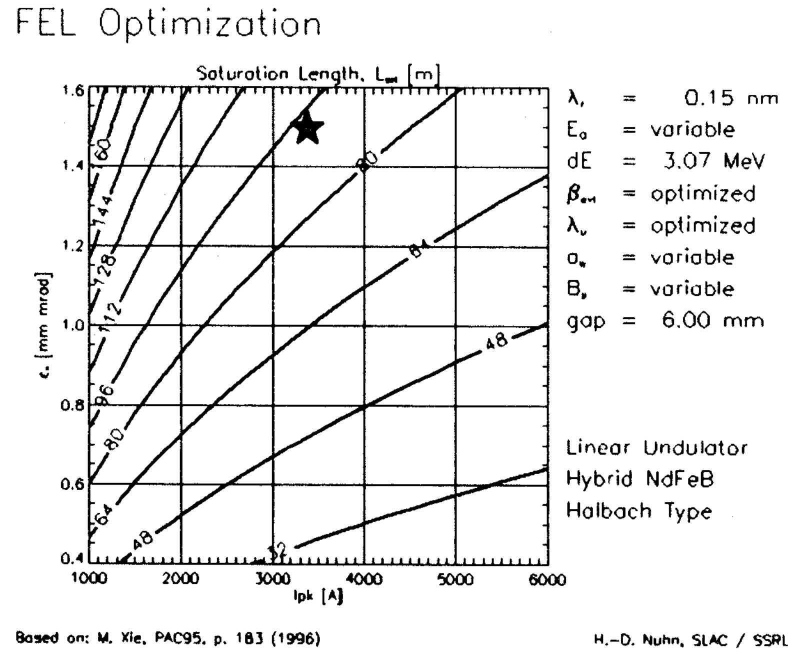
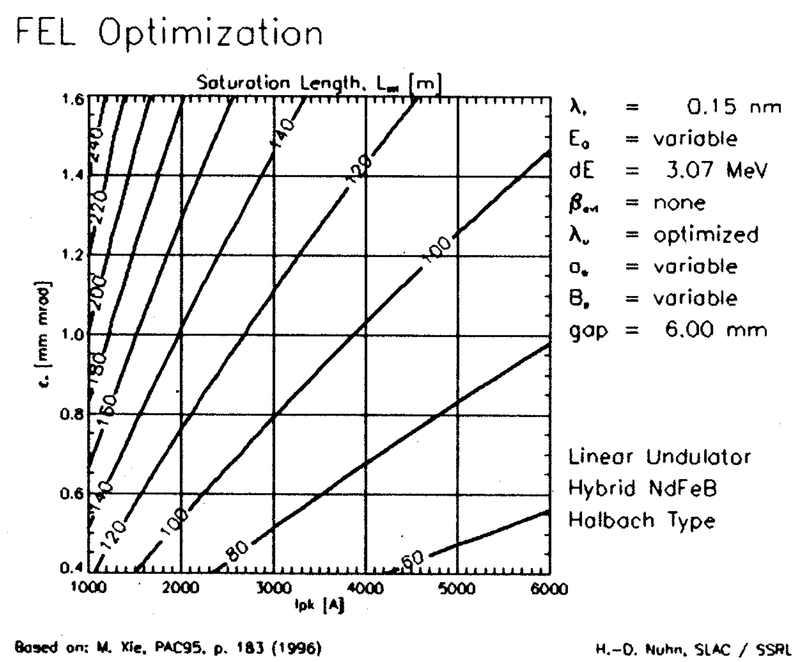
Optimization Results - L_{sat}

$\lambda_r = 0.15$ nm
linear

★ LCLS





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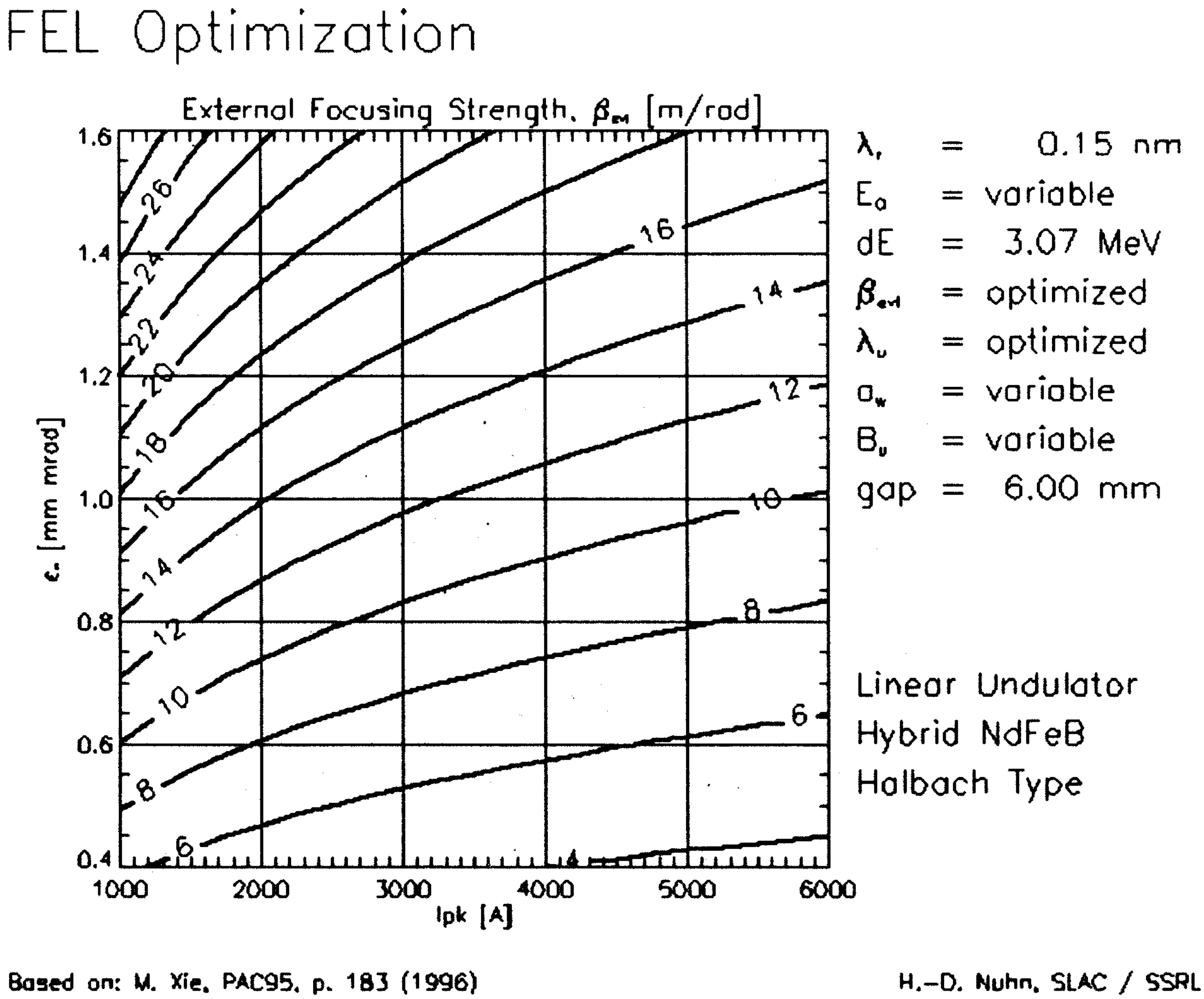
Optimization Results - Focusing

λ_r = 0.15 nm
linear



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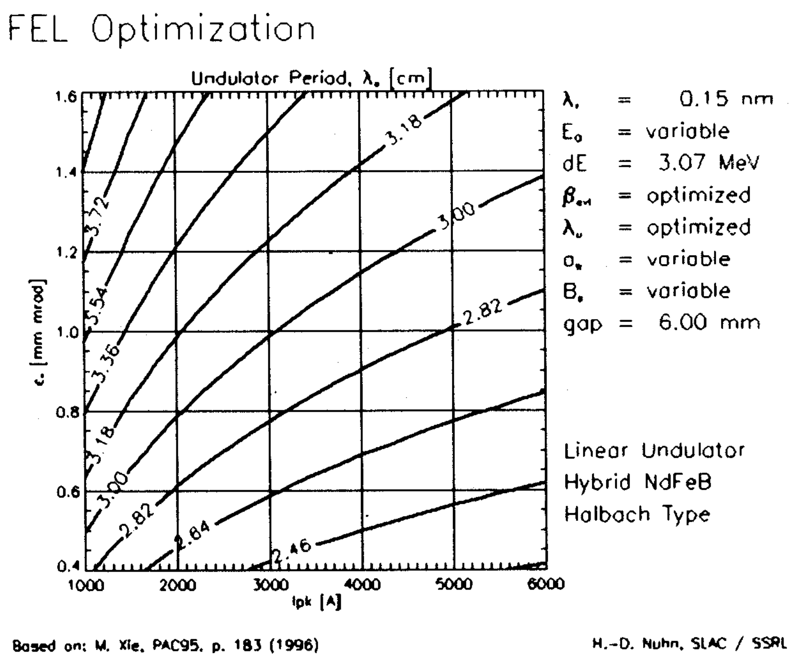
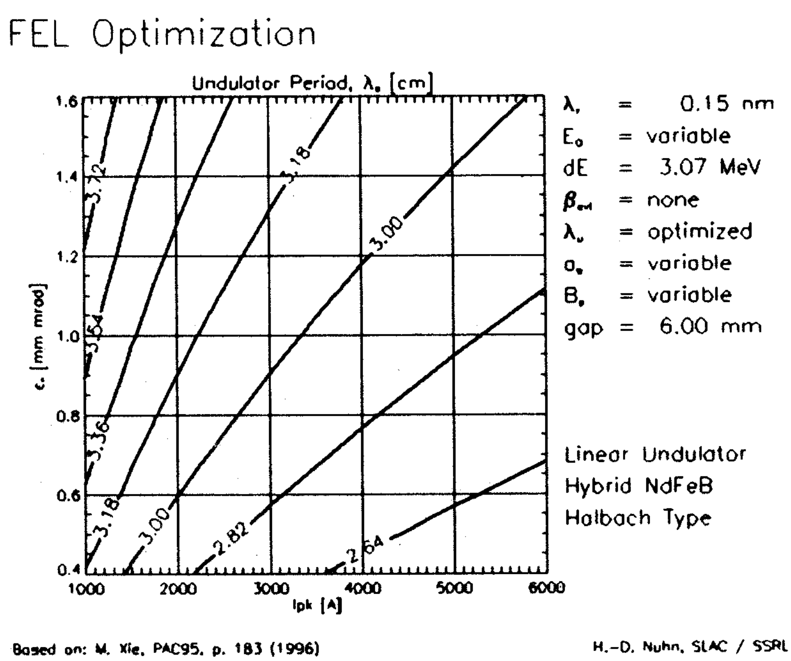
Optimization Results - λ_u

$\lambda_r = 0.15$ nm

linear





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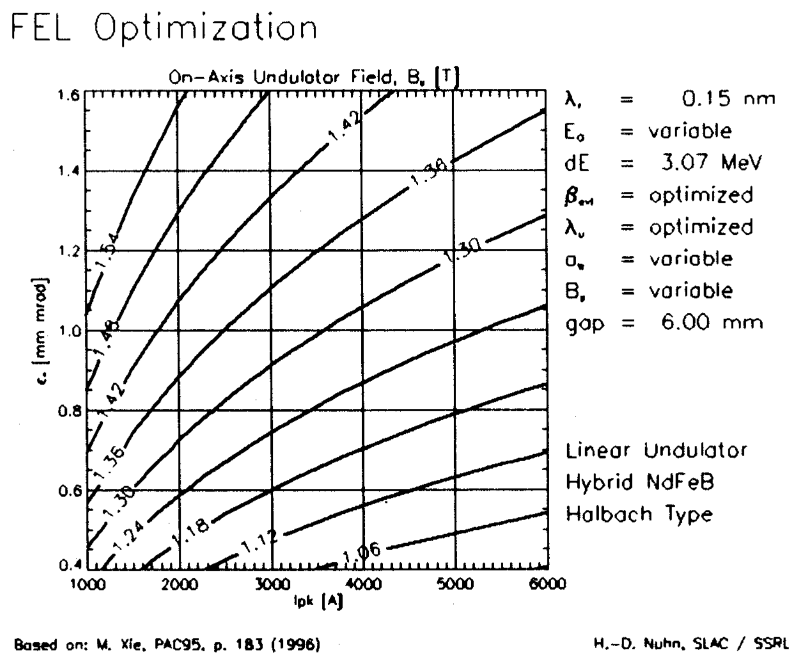
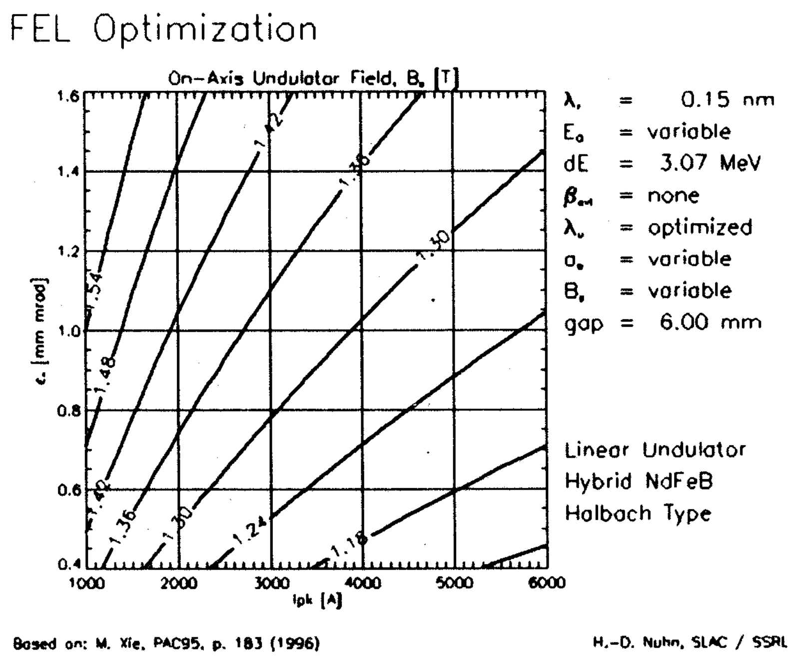
Optimization Results - B_u

$\lambda_r = 0.15$ nm
linear





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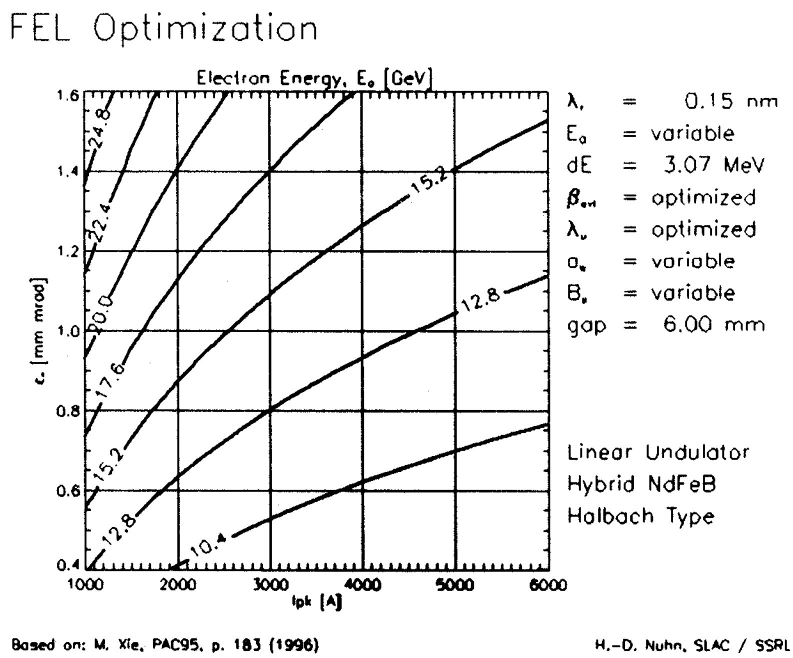
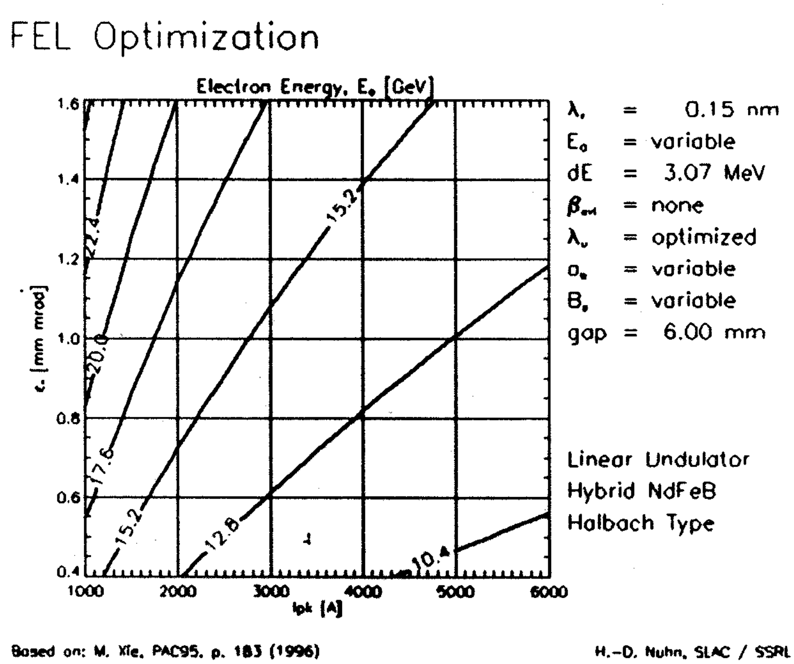
Optimization Results - Energy

λ_r = 0.15 nm
linear





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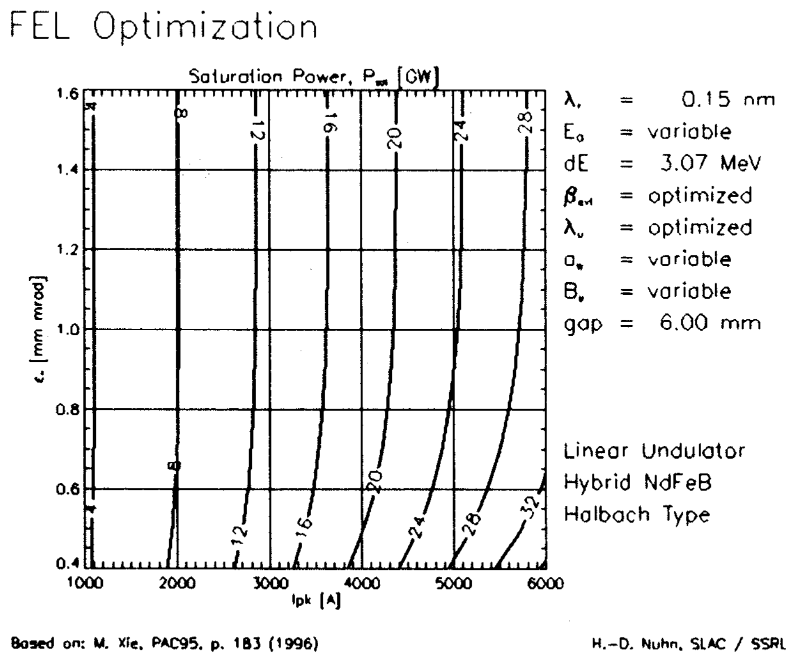
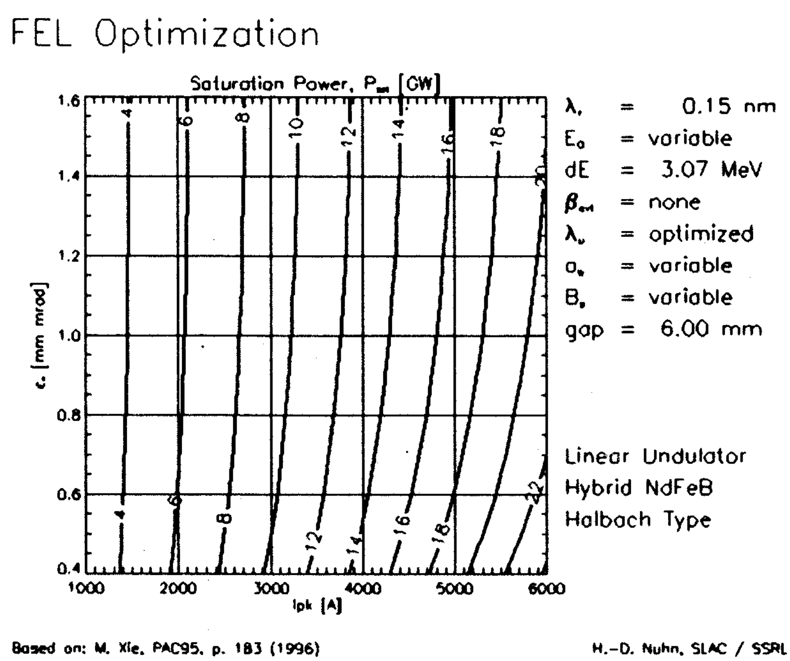
Optimization Results - P_{sat}

$\lambda_r = 0.15$ nm
linear





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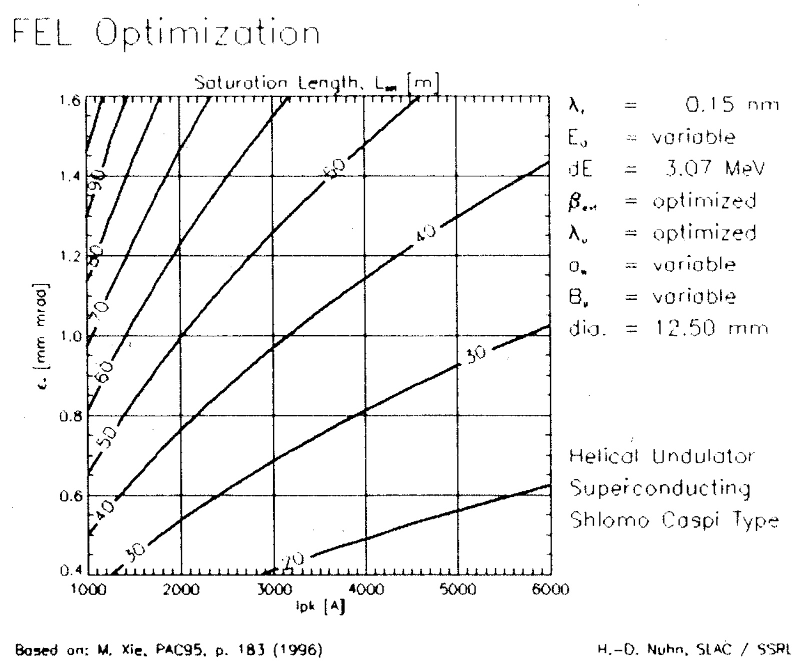
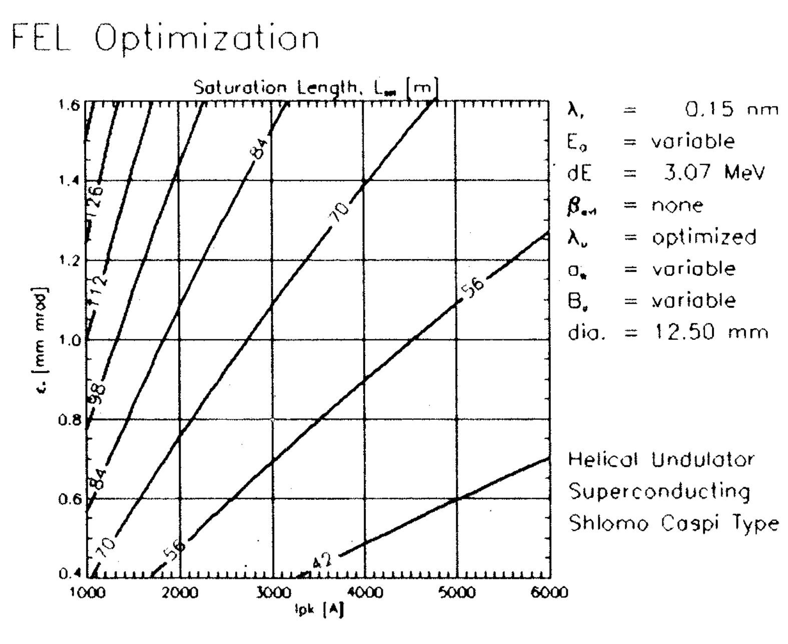
Optimization Results - L_{sat}

λ_r = 0.15 nm
helical





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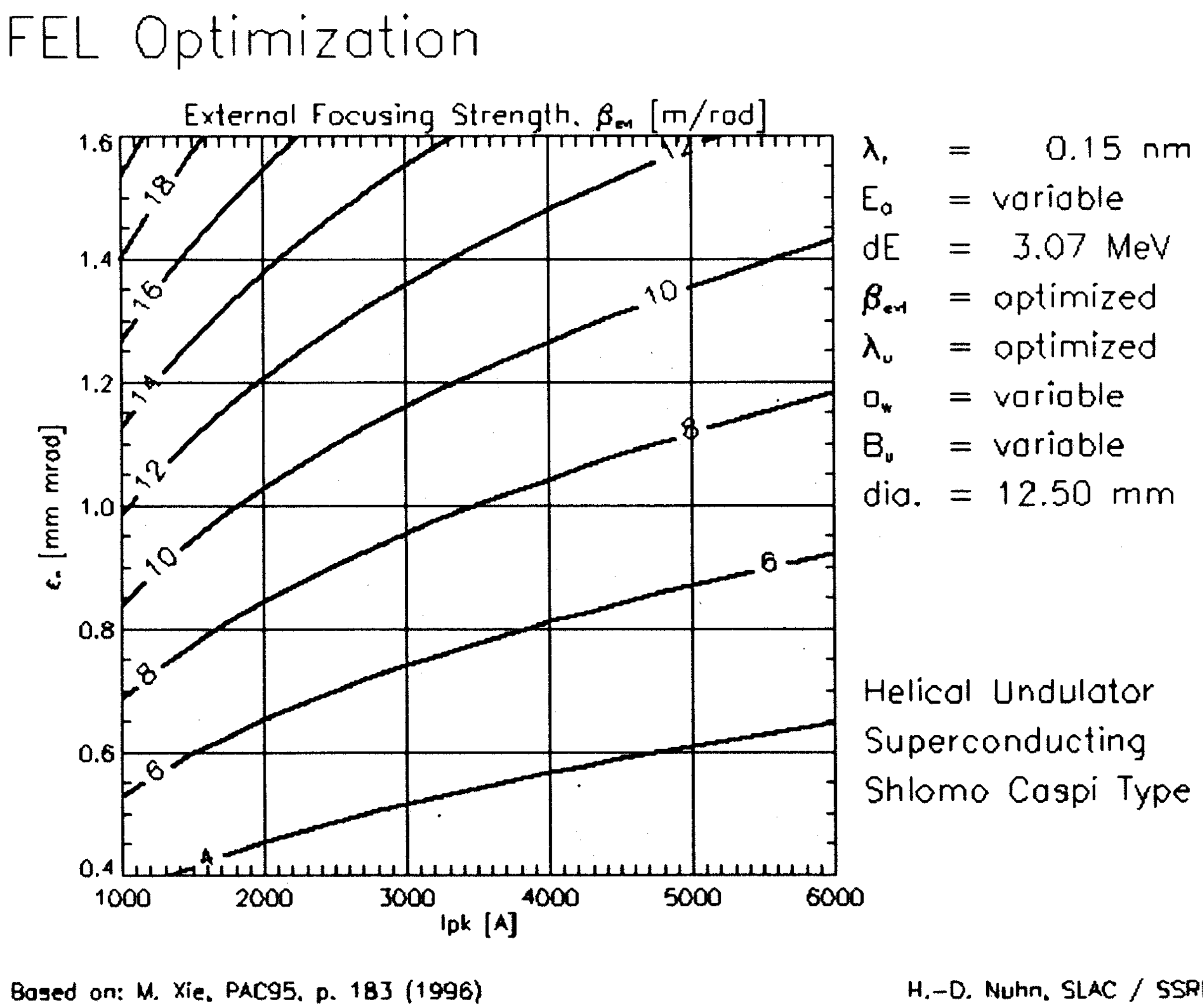
Optimization Results - Focusing

λ_r = 0.15 nm
helical



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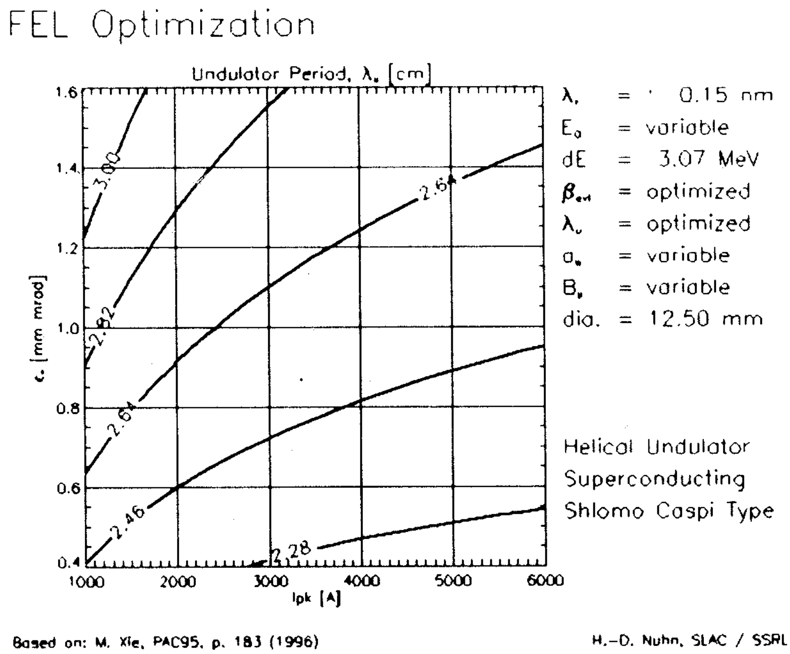
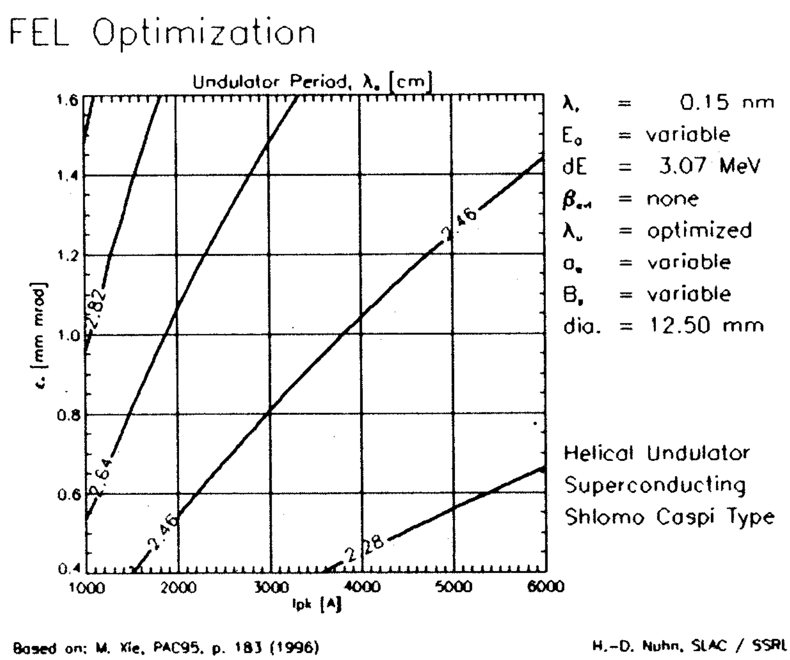
Optimization Results - λ_u

$\lambda_r = 0.15$ nm
helical





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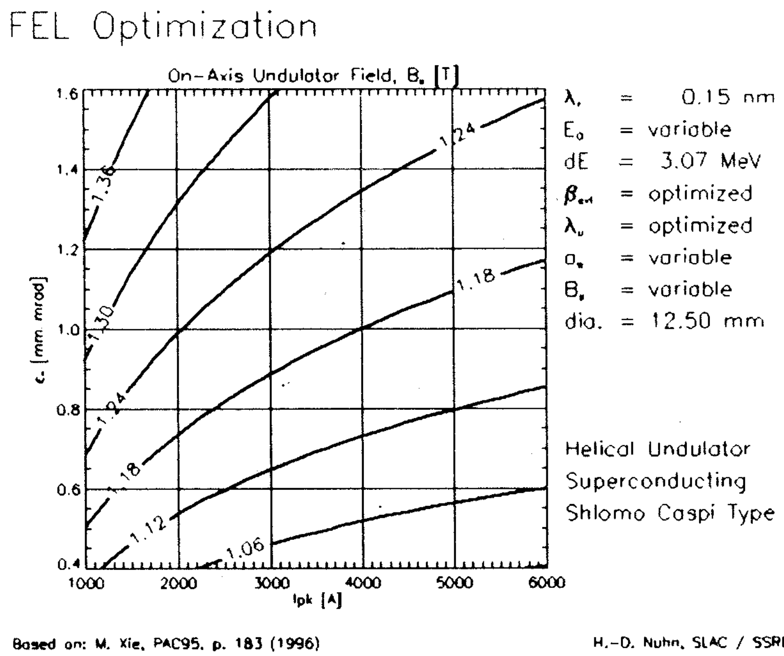
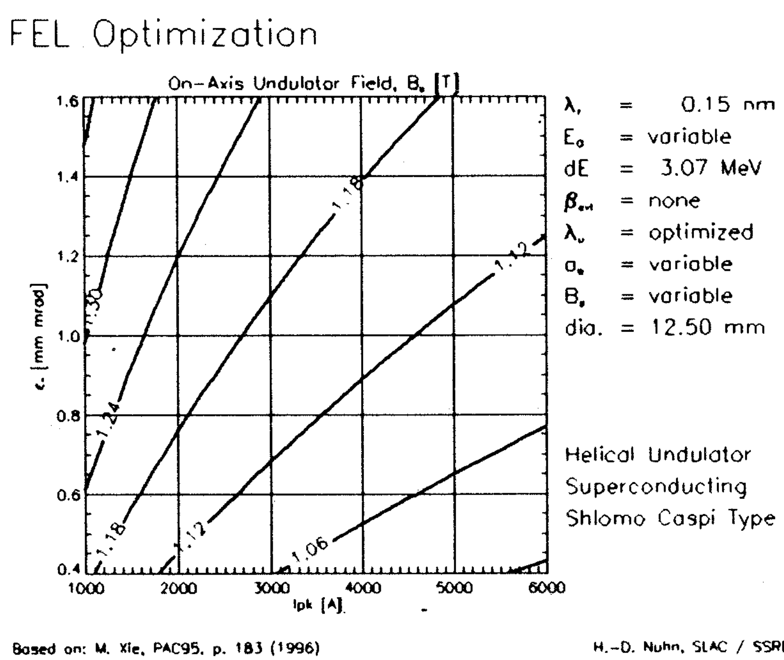
Optimization Results - B_u

λ_r = 0.15 nm
helical





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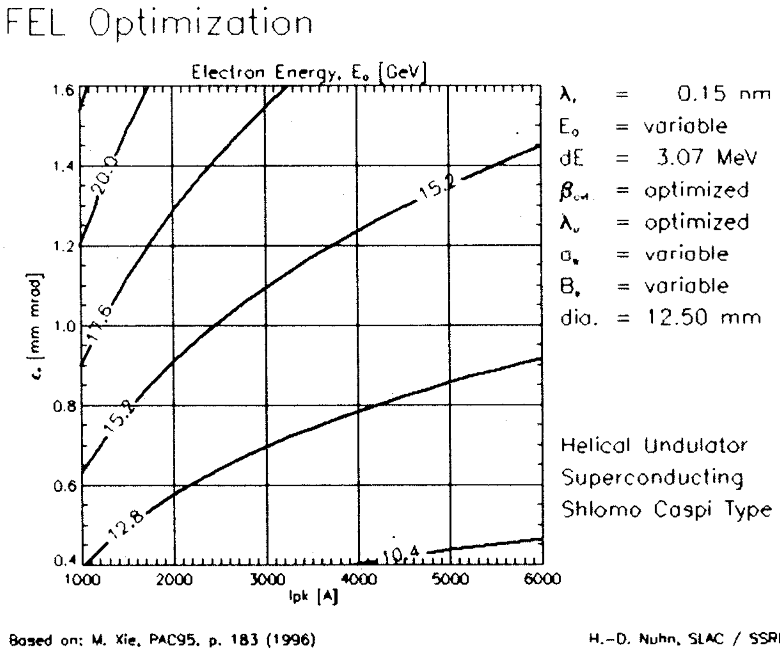
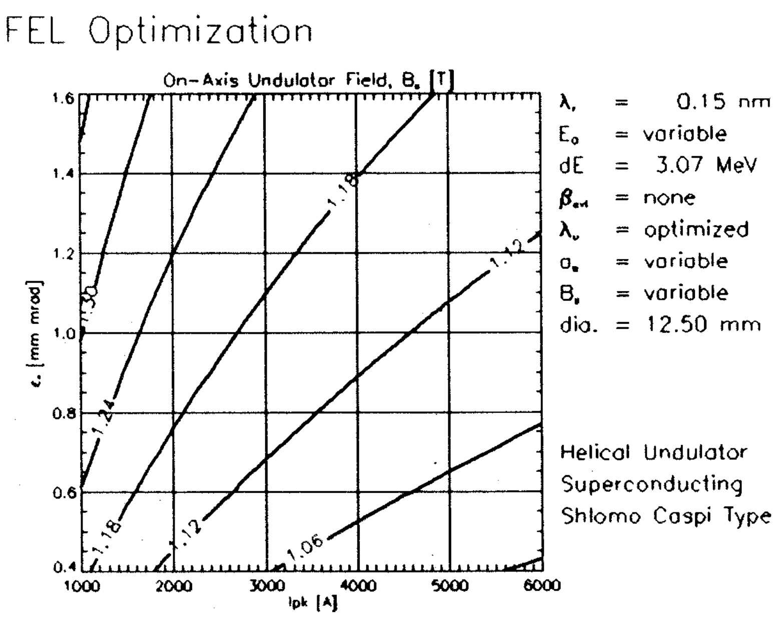
Optimization Results - Energy

λ_r = 0.15 nm
helical





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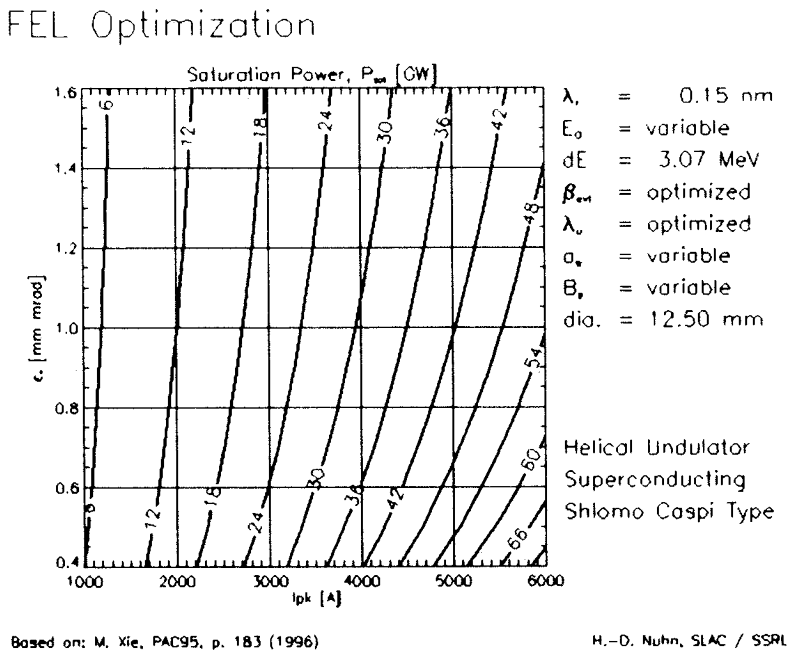
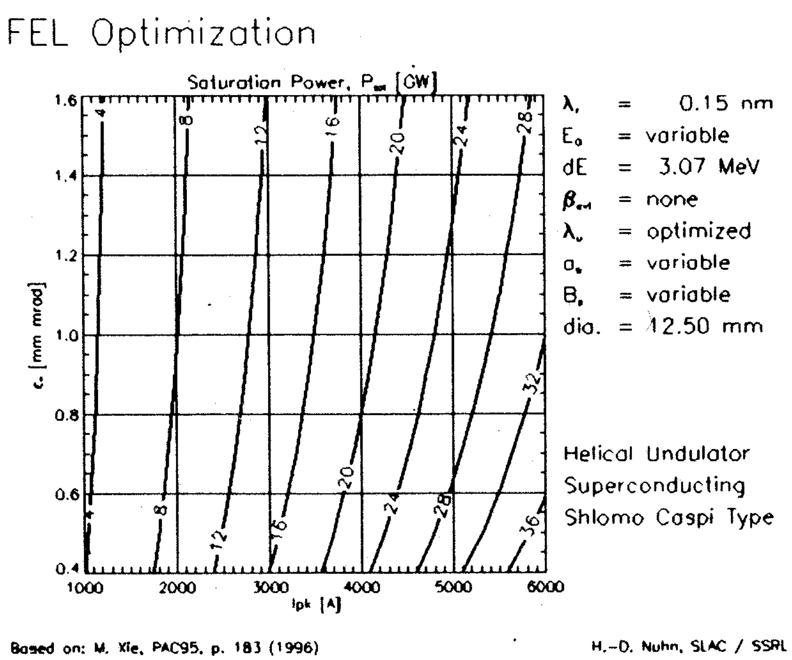
Optimization Results - P_{sat}

λ_r = 0.15 nm
helical





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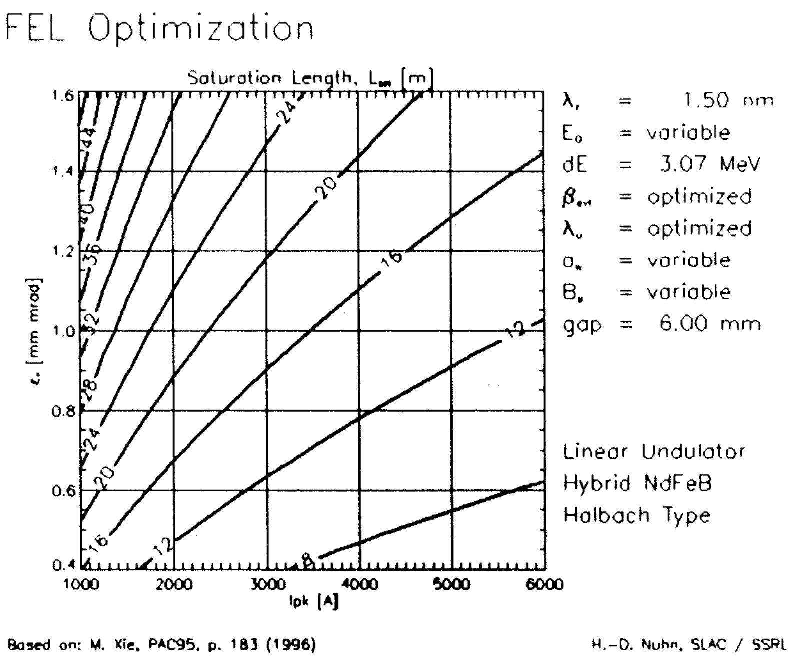
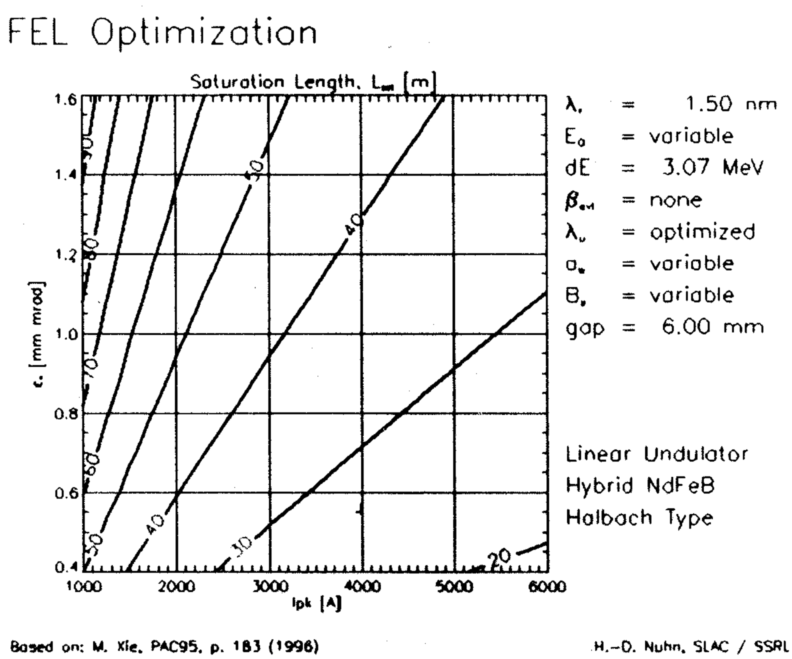
Optimization Results - L_{sat}

λ_r = 1.5 nm
linear





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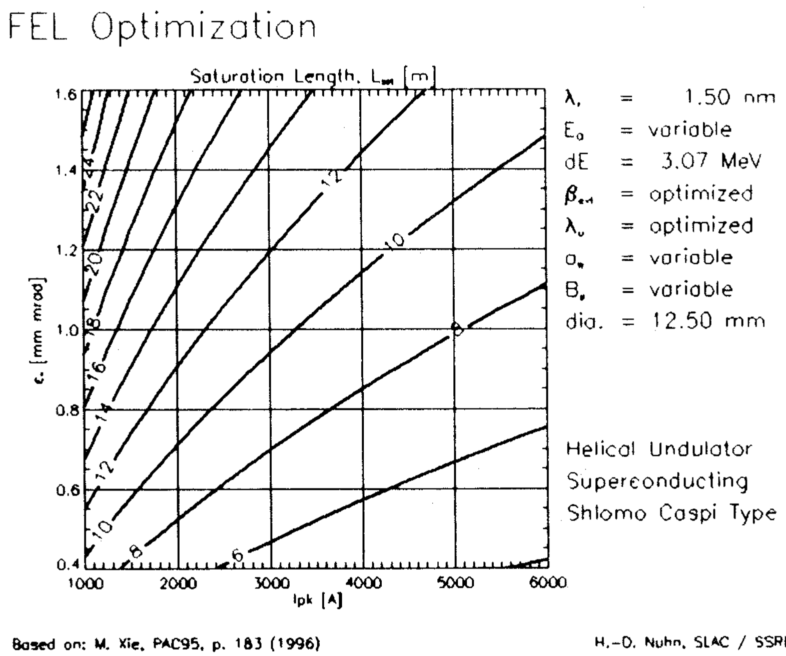
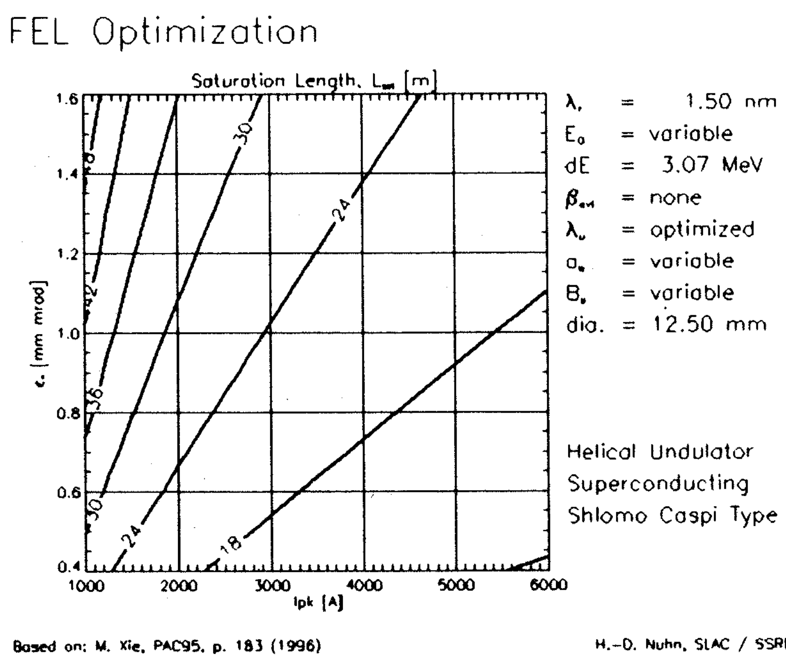
Optimization Results - L_{sat}

λ_r = 1.5 nm
helical





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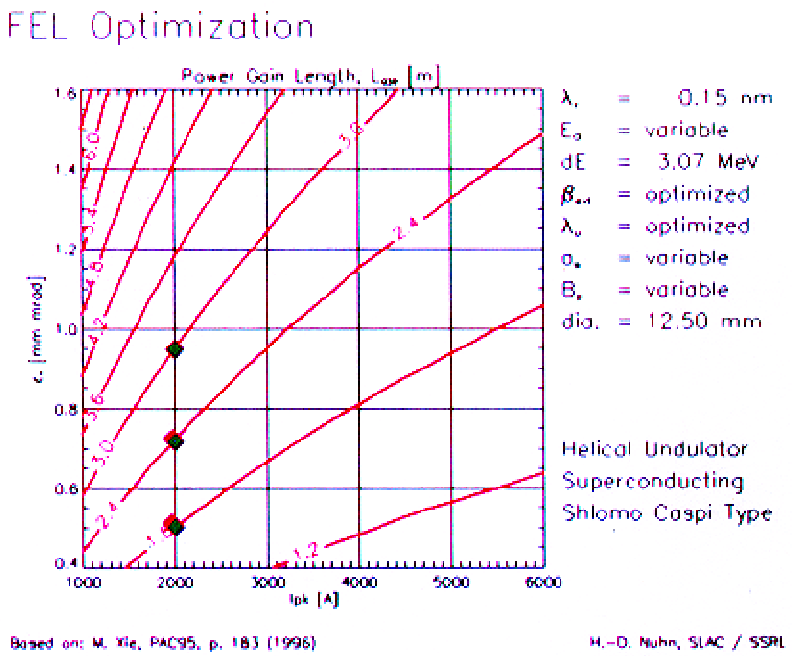
Comparison with Simulations

$\lambda_r = 0.15$ nm

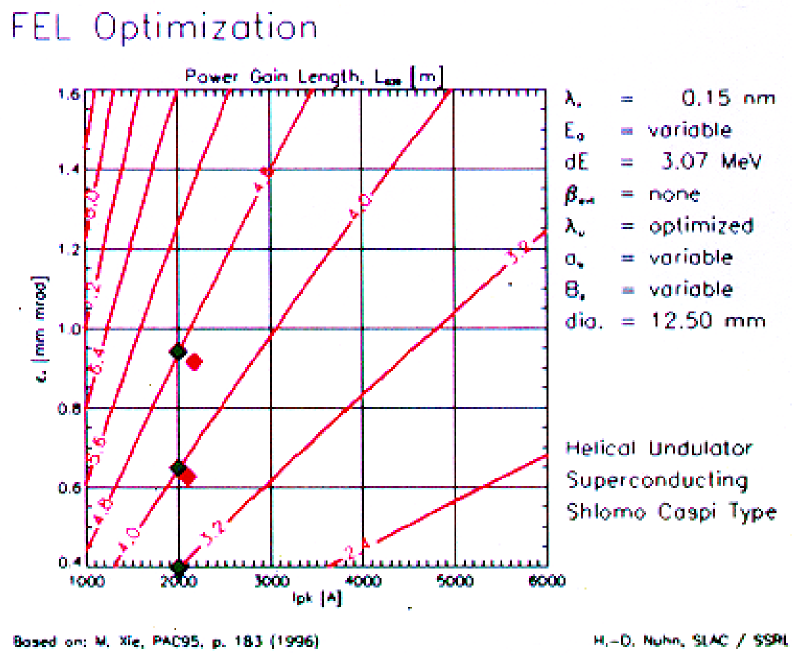


GINGER Simulation Results:

ε_n = 0.950 mm mrad, L_G = 3.070 m

ε_n = 0. 724 mm mrad, L_G = 2.426 m

ε_n = 0.504 mm mrad, L_G = 1.876 m



GINGER Simulation Results:

ε_n = 0.936 mm mrad, L_G = 4.571 m

ε_n = 0.645 mm mrad, L_G = 3.885 m

ε_n = 0.398 mm mrad, L_G = 3.208 m

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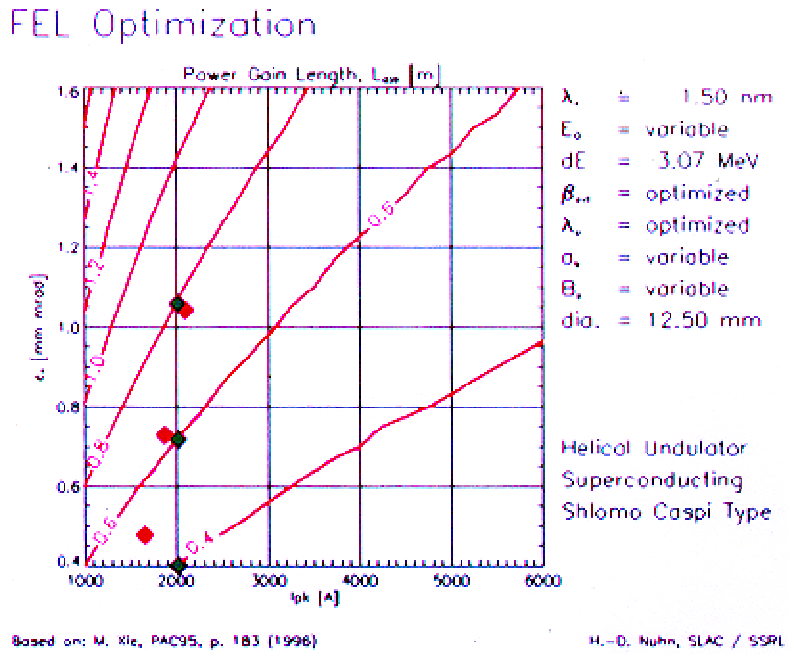
Comparison with Simulations

$\lambda_r = 1.5$ nm

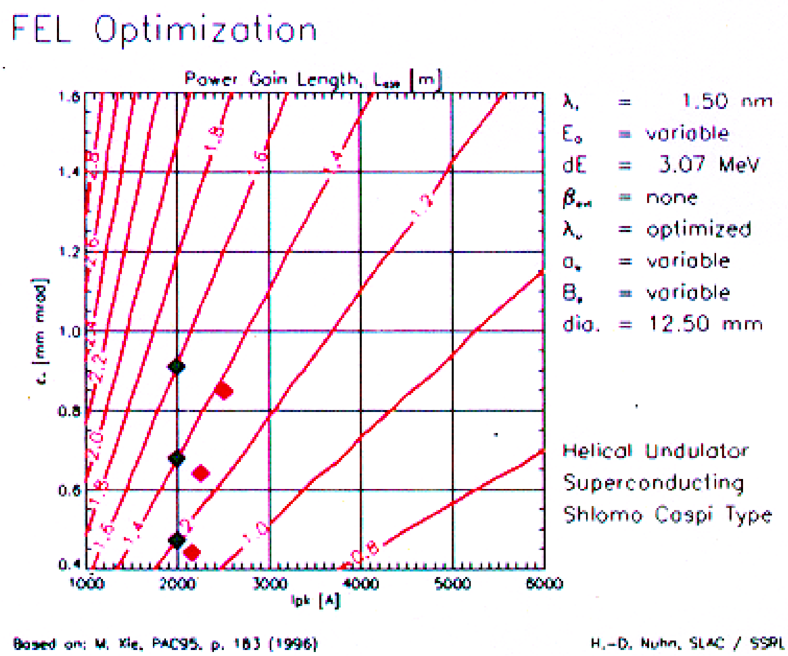


GINGER Simulation Results:

ε_n = 1.059 mm mrad, L_G = 0.783 m

ε_n = 0. 718 mm mrad, L_G = 0.640 m

ε_n = 0.402 mm mrad, L_G = 0.516 m



GINGER Simulation Results:

ε_n = 0.913 mm mrad, L_G = 1.375 m

ε_n = 0.678 mm mrad, L_G = 1.281 m

ε_n = 0.476 mm mrad, L_G = 1.122 m

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 3.06, ε_n=0.95 mm mrad, I_{pk}=2000A, λ_r=1.5 Å,
λ_u= 2.66 cm, γ=30283, σ_γ = 6, β_{xy}=9.1 m, L_{sat} = 48.1 m, P_{sat} = 12.1 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	0.54	-0.61	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.61	-0.85	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	6.68	-2.74	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.26	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.50	1.06	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	-1.08	1.23	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.13	-0.47	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-4.58	2.42	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.04	0.68	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-0.29	0.06		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-1.52	0.85		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.86	-0.50		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}	0.42	0.00		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	

ba

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.48, ε_n=0.5 mm mrad, I_{pk}=2000A, λ_r=1.5 Å,
λ_u= 2.4 cm, γ=23906, σ_γ = 6, β_{xy}=4.5 m, L_{sat} = 28.5 m, P_{sat} = 14.5 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const
λ_r	0.49	-0.59	γ	λ_u γ g a_w σ_γ ε_n I_{pk} β_{ext}
λ_r	1.53	-0.82	g a_w	λ_u γ σ_γ ε_n I_{pk} β_{ext}
λ_u	6.58	-2.76	λ_r	γ g a_w σ_γ ε_n I_{pk} β_{ext}
λ_u	4.28	0.00	γ	λ_r g a_w σ_γ ε_n I_{pk} β_{ext}
λ_u	-0.56	1.09	g a_w	λ_r γ σ_γ ε_n I_{pk} β_{ext}
γ	-0.98	1.18	λ_r	λ_u g a_w σ_γ ε_n I_{pk} β_{ext}
γ	2.07	-0.47	g a_w	λ_r λ_u σ_γ ε_n I_{pk} β_{ext}
g	-4.55	2.45	λ_r a_w	λ_u γ σ_γ ε_n I_{pk} β_{ext}
g	-3.08	0.69	γ a_w	λ_r λ_u σ_γ ε_n I_{pk} β_{ext}
σ_γ	-0.24	0.04		λ_r λ_u γ g a_w ε_n I_{pk} β_{ext}
ε_n	-1.40	0.80		λ_r λ_u γ g a_w σ_γ I_{pk} β_{ext}
I_{pk}	1.84	-0.49		λ_r λ_u γ g a_w σ_γ ε_n β_{ext}
β_{ext}	0.43	0.00		λ_r λ_u γ g a_w σ_γ ε_n I_{pk}

bc

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.93, ε_n=0.936 mm mrad, I_{pk}=2000A, λ_r=1.5 Å,
λ_u= 2.60 cm, γ=28824, σ_γ = 6, β_{xy}=none, L_{sat} = 77.9 m, P_{sat} = 8.0 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	-0.01	-0.46	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.14	-0.75	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.76	-2.10	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.80	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.40	1.30	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	0.02	0.93	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.31	-0.58	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.29	2.17	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.32	0.83	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-1.09	0.41		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-0.74	0.51		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.76	-0.46		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}				λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.47, ε_n=0.398 mm mrad, I_{pk}=2000A, λ_r=1.5 Å, λ_u= 2.39 cm, γ =23810, σ_γ = 6, β_{xy}=none, L_{sat} = 51.6 m, P_{sat} = 9.5 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	-0.05	-0.45	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.07	-0.73	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.54	-2.09	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.78	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.45	1.34	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	0.10	0.89	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.24	-0.57	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.18	2.18	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.33	0.85	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-1.01	0.38		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-0.62	0.46		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.72	-0.44		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}												

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.47, ε_n=0.398 mm mrad, I_{pk}=2000A, λ_r=1.5 Å, λ_u= 2.39 cm, γ=23810, σ_γ = 6, β_{xy}=none, L_{sat} = 51.6 m, P_{sat} = 9.5 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	-0.05	-0.45	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.07	-0.73	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.54	-2.09	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.78	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.45	1.34	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	0.10	0.89	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.24	-0.57	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.18	2.18	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.33	0.85	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-1.01	0.38		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-0.62	0.46		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.72	-0.44		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}												

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.74, ε_n=1.06 mm mrad, I_{pk}=2000A, λ_r=15.0 Å, λ_u= 2.51 cm, γ =8438, σ_γ = 6, β_{xy}=1.3 m, L_{sat} = 13.5 m, P_{sat} = 11.1 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	0.42	-0.56	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.47	-0.80	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	6.17	-2.58	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.24	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.54	1.08	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	-0.84	1.13	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.09	-0.47	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-4.28	2.33	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.05	0.69	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-0.42	0.13		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-1.35	0.78		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.92	-0.53		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}	0.42	0.00		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 3.02, ε_n=0.913 mm mrad, I_{pk}=2000A, λ_r=15.0 Å, λ_u= 2.64 cm, γ =9435, σ_γ = 6, β_{xy}=none, L_{sat} = 27.7 m, P_{sat} = 7.6 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	-0.11	-0.42	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.05	-0.72	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.28	-1.90	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.76	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.42	1.31	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	0.22	0.85	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.32	-0.59	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.00	2.04	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.30	0.84	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-1.28	0.51		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-0.63	0.46		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.84	-0.50		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}												

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.09, ε_n=0.402 mm mrad, I_{pk}=2000A, λ_r=15.0 Å, λ_u= 2.2 cm, γ=6300, σ_γ = 6, β_{xy}=0.45 m, L_{sat} = 6.6 m, P_{sat} = 13.9 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	0.36	-0.54	γ		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.37	-0.77	g a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	5.97	-2.58	λ_r			γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.22	0.00	γ	λ_r			g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.62	1.13	g a_w	λ_r		γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	-0.73	1.07	λ_r		λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.01	-0.47	g a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
g	-4.20	2.36	λ_r a_w		λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
g	-3.08	0.72	γ a_w	λ_r	λ_u				σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-0.31	0.08		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-1.17	0.70		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.87	-0.51		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}	0.42	0.00		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	

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Sensitivities to Parameter Changes

g = 12.5 mm, a_w = 2.69, ε_n=0.476 mm mrad, I_{pk}=2000A, λ_r=15.0 Å, λ_u= 2.49 cm, γ =8278, σ_γ = 6, β_{xy}=none, L_{sat} =20.7 m, P_{sat} = 8.6 GW

Sensitivity of Saturation Power and Length Parameter x: Helical Undulator.

$$\frac{\Delta P_{sat} / P_{sat}}{\Delta x / x} \qquad \frac{\Delta L_{sat} / L_{sat}}{\Delta x / x}$$

	P_{sat}	L_{sat}	var	const								
λ_r	-0.12	-0.42	γ	λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_r	1.01	-0.70	g a_w	λ_r	λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.18	-1.91	λ_r		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	4.76	0.00	γ	λ_r	λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
λ_u	-0.43	1.32	g a_w	λ_r	λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
γ	0.25	0.83	λ_r		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
γ	2.26	-0.58	g a_w	λ_r	λ_u	γ			σ_γ	ε_n	I_{pk}	β_{ext}
gap	-2.94	2.06	λ_r		λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
gap	-3.31	0.84	γ	λ_r	λ_u		g	a_w	σ_γ	ε_n	I_{pk}	β_{ext}
σ_γ	-1.21	0.48		λ_r	λ_u	γ	g	a_w		ε_n	I_{pk}	β_{ext}
ε_n	-0.57	0.44		λ_r	λ_u	γ	g	a_w	σ_γ		I_{pk}	β_{ext}
I_{pk}	1.82	-0.49		λ_r	λ_u	γ	g	a_w	σ_γ	ε_n		β_{ext}
β_{ext}				λ_r	λ_u	γ	g	a_w	σ_γ	ε_n	I_{pk}	

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Conclusion

- **FEL scenarios have been analyzed for an emittance range from 1.6 to 0.4 mm mrad using 3D analytic theory.**
- **Results compare well with multidimensional, polychromatic FEL simulation code, GINGER.**
- **Lower emittance reduces undulator length and energy requirements.**
- **Lower emittance can also be utilized to reduce peak current.**
- **Elimination of quadrupole focusing becomes an interesting option.**
- **Dependence of trajectory tolerances on smaller emittance is still to be studied.**

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