Longitudinal Space Charge Instability

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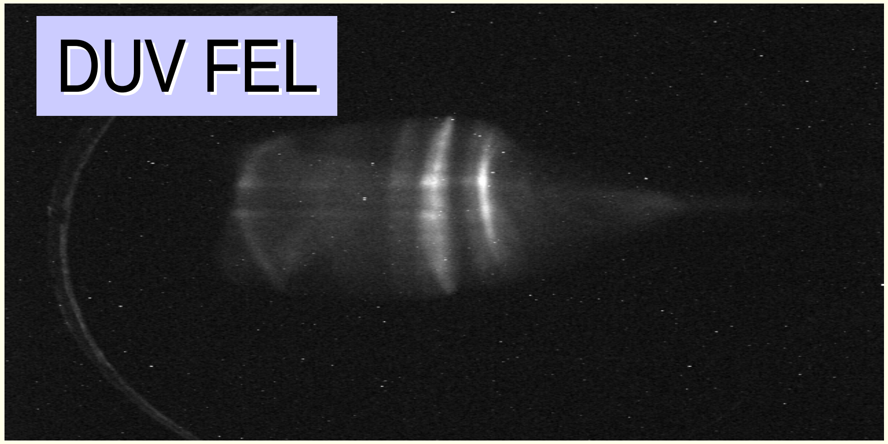
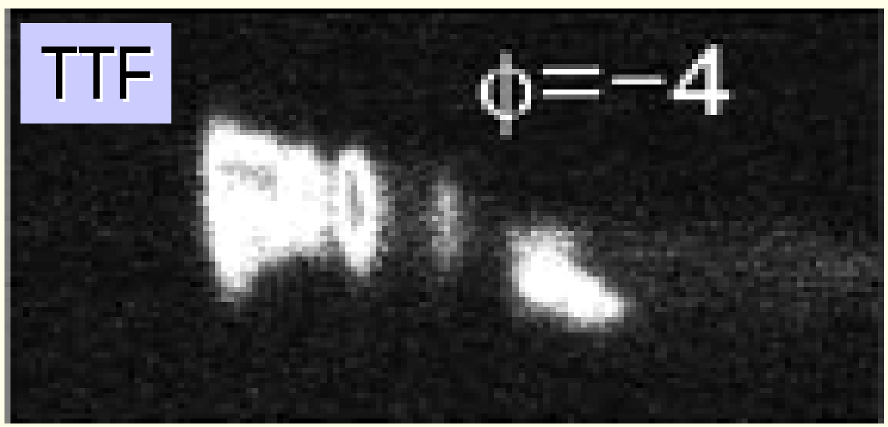
T.Shaftan, BNL

September 24, 2003

- ***LSC observed experimentally***
- ***Theory***
- ***Application to LCLS***
- ***Comparison theory with Simulations***





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Motivations

- *Experimental*
  - *TTF observations*

M. Huning et al., NIM A 475 (2001) p. 348

  - *DUVFEL observations*

W.S. Graves, et al., PAC 2001, p. 2860

  - *T.Shaftan et al. experiments at the DUVFEL*

T.Shaftan et al., PAC03, "Micro-bunching and beam break-up in the DUV FEL accelerator", also in FEL03

with modeling of mechanism discussed in March at SLAC

Z.Huang, T.Shaftan, SLAC-PUB-97, May 2003

- *Theory*
  - *Saldin* , Schneidmiller, Yurkov, "Longitudinal Space Charge Driven MicroBunching Instability in TTF2 Linac" TESLA-FEL-2003-02,May 2003

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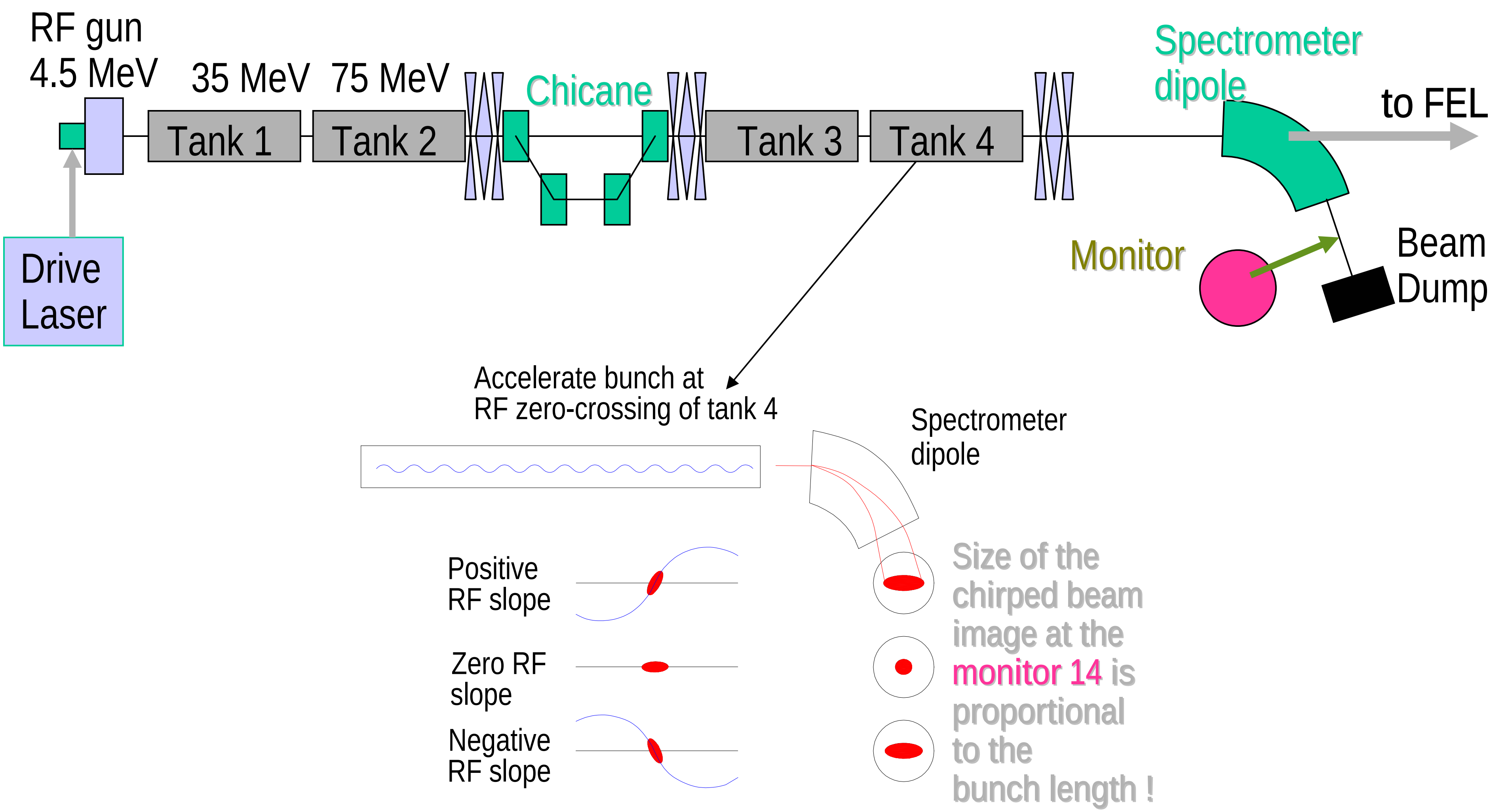
The DUVFEL Accelerator



The DUVFEL Accelerator

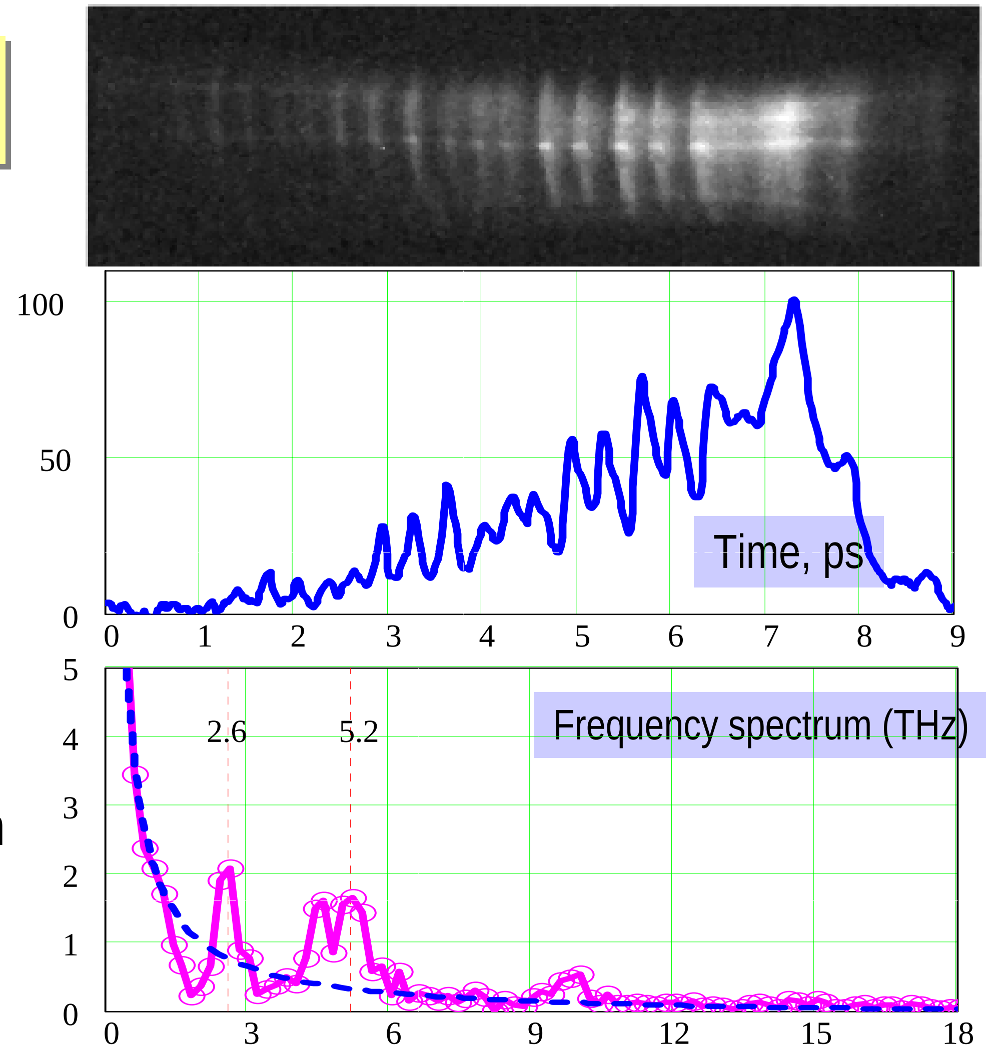
Structure with a Large Number Spikes

• “Zero-phasing” image of uncompressed bunch with a large number of sharp spikes

• Energy spectrum, derived from the image, horizontal axis is scaled in picoseconds

<u>Time or Energy</u>?

• Frequency spectrum of upper plot. Spectrum shows modulation with harmonics in THz range. Harmonics ←→ sharpness of the spikes.



Modulation dynamics with Compression (~300 pC)

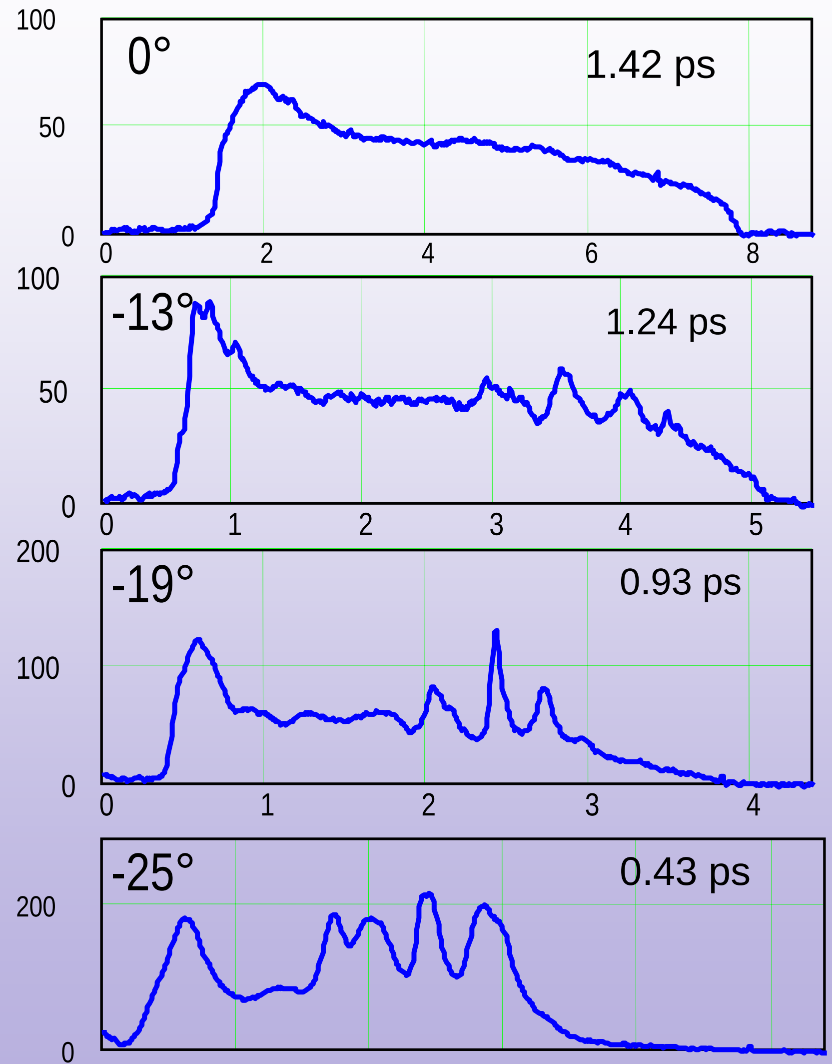
• Dynamics at "high" (>200 pC) charge differs from the dynamics at "low" charge

• Uncompressed bunch profile is smooth

• Experiment on modulation dynamics:
Keep chicane constant and increase chirping tank phase (0→-13→-19→-25 degrees)

Modulation shows up during compression Process

■Compression: a} decreases modulation period (C times); b) increases bunch peak current (C times)



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This is not CSR!

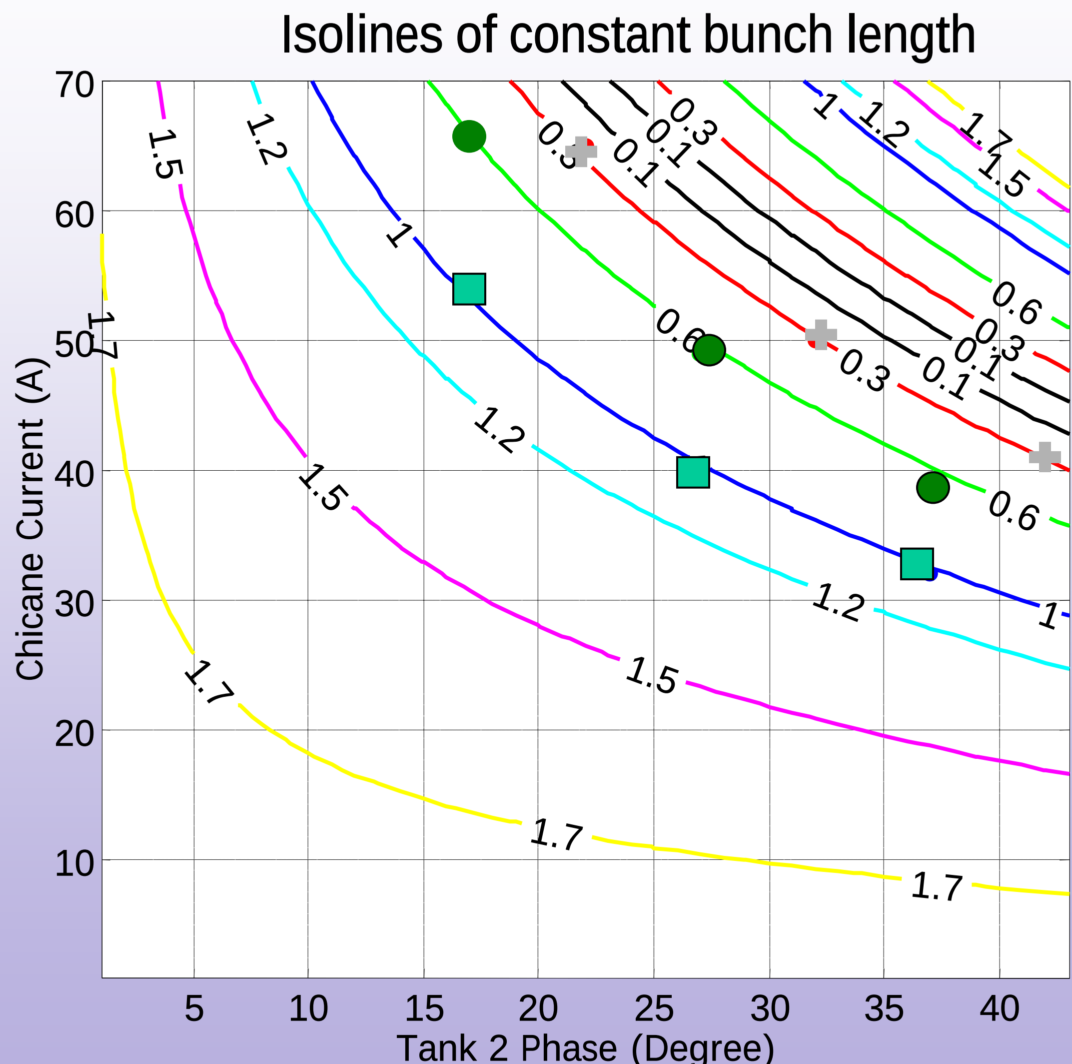
This is not CSR!

- *CSR-related effect ? → should be sensitive to the bending radius in the chicane magnets*

- *Experiment:*
  final bunch length maintained constant while chicane strength changed

Compression factor is 1-hR56 → there is always a combination of h and R56 that will maintain a constant compression ratio

- *Result of the experiment:*
  in general the modulation is insensitive to the chicane settings



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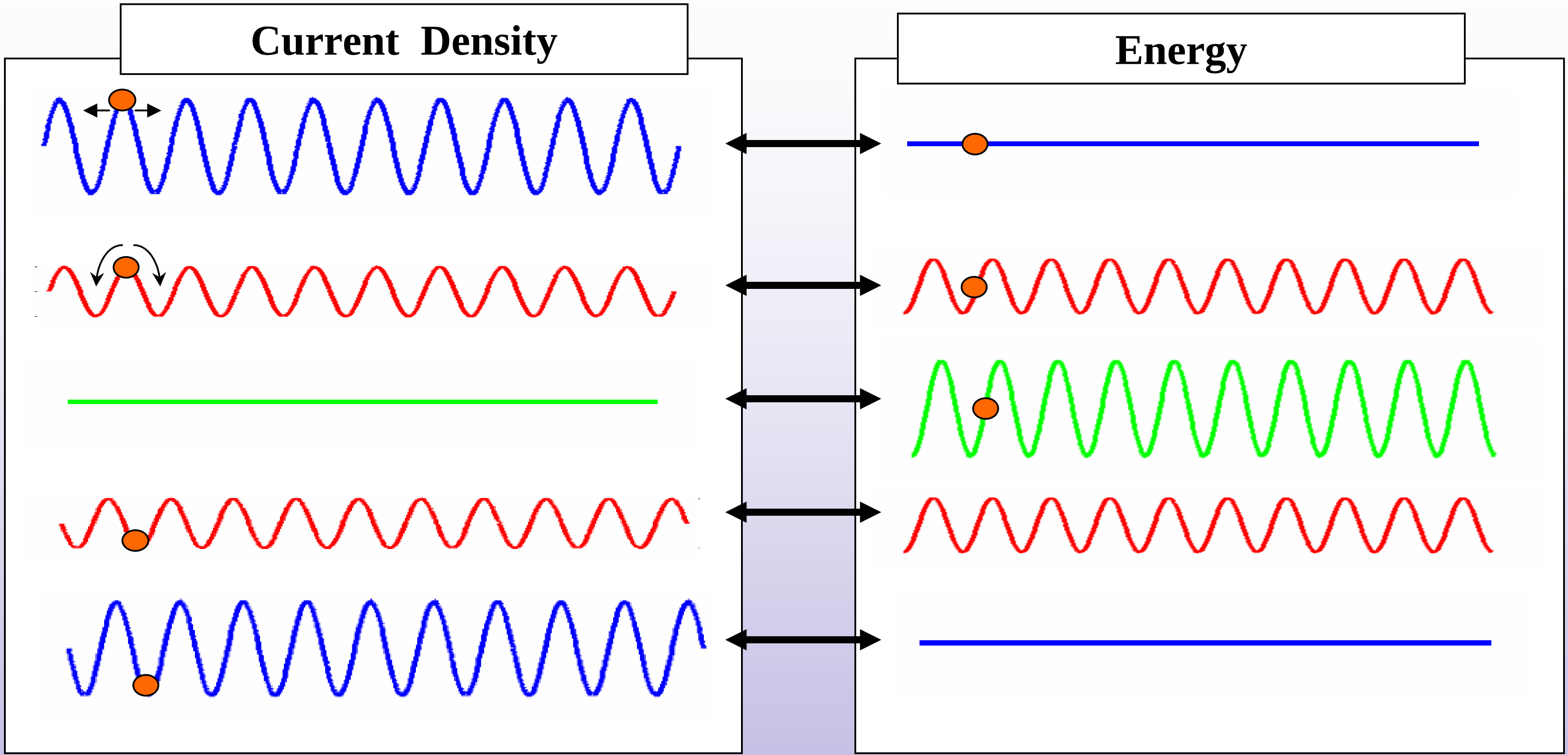
Plasma Oscillation in a Coasting Beam



•The self-consistent solution is the space charge oscillation

$$\Omega = c\left[\frac{I_0}{\gamma^3 I_A} k|Z(k)|\right]^{1/2} = \text{plasma freq.}\quad k \to \infty$$

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Longitudinal Space Charge Impedance

- **For a round, parallel electron beams with a uniform transverse cross section of radius r_b, the on-axis longitudinal space charge impedance is (cgs units)**

$$Z(k) = \frac{4i}{kr_b^2}\left[1 - \frac{kr_b}{\gamma}K_1\left(\frac{kr_b}{\gamma}\right)\right]$$

$$\approx \frac{4i}{kr_b^2} \text{ if } \frac{kr_b}{\gamma} \gg 1$$ (pancake beam)

$$\approx \frac{ik}{\gamma^2}\left(1 + 2\ln\frac{\gamma}{r_b k}\right) \text{ if } \frac{kr_b}{\gamma} \ll 1$$ (pencil beam)

- **Free space approximation is good when $\gamma\,\lambda/(2\pi)$ << beam pipe radius**
- **Off-axis LSC is smaller and can increase the energy spread**

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Integral Equation

- Bunching parameter b at modulation wavelength λ = 2π/k

$$I = I_o(1 + b\cos(kz))$$

- Linear evolution of b(k;s) can be described by an integral equation (Heifets, Stupakov, Krinsky; Huang & Kim)

$$b(k(s);s) = b_0(k(s);s) + \int_0^s d\tau K(\tau,s) b(k(\tau);\tau)$$

$$\text{kernel } K(\tau,s) = ik(s) R_{56}(\tau \to s) \frac{I(\tau)}{\gamma I_A} Z(k(\tau)) \times \underbrace{\exp(\ldots\varepsilon,\sigma_\delta\ldots)}_{\text{Landau damping}}$$

- For arbitrary initial condition (density and/or energy modulation), this determines the final microbunching

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Including acceleration

- beam energy $\gamma_r(s)$ increases in the linac. Generalize the momentum compaction $R_{56}'(\tau \to s)$ as the path length change at s due to a small change in γ (not δ) at τ:

$$R'_{56}(\tau \to s) = \int_\tau^s \frac{ds'}{\gamma_r(s)^3} \qquad \text{for } \gamma_r \gg 1$$

- The integral equation for LSC microbunching in the linac is

$$b(k,s) = b_0(k,s) + ik \int_0^s d\tau R'_{56}(\tau \to s) \frac{I_0}{I_A} Z(k,\tau) b(k,\tau)$$

- In a drift, $R'_{56} = \frac{s-\tau}{\gamma^3} = \frac{R_{56}}{\gamma}$

$$\frac{d^2 b(k,s)}{ds^2} = -\Omega^2 b(k,s)$$ ➔ Space charge oscillation

- When $\gamma \to 1$, $R_{56}'=0$, $b(k,s)=b_0(k,s)$, beam is "frozen"

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Space Charge Oscillation – 2 regimes

- Density and energy modulation in a drift at distance s

$$b(k;s) = b_0(k)\cos(\Omega s/c),$$

$$\Delta\gamma(k;s) = -\frac{I_0}{I_A} Z(k) b_0(k) \frac{\sin(\Omega s/c)}{\Omega/c}$$

- At a very large γ, plasma phase advance (Ωs/c) ¿ 1, beam is “frozen,” energy modulation gets accumulated (Saldin/Schneidmiller/Yurkov, TESLA-FEL-2003-02)

$$b(k;s) \approx b_0(k)$$

$$\Delta\gamma(k;s) \approx -\frac{I_0}{I_A} Z(k) b_0(k) s$$

LSC acts like a normal impedance at high energies

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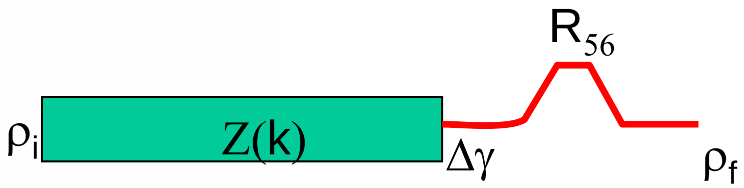
Klystron Amplitfication mechanism

Saldin, Schneidmiller, Yurkov



Initial modulation

$$I(z) = I_0 \left[1 + \rho_{\mathrm{i}} \cos(kz)\right]$$

Wakefields:

$$\Delta\gamma = \frac{|Z(k)|}{Z_0} \frac{I_0}{I_{\mathrm{A}}} \rho_{\mathrm{i}}$$

$$Z_0 = 377\ \Omega\ , \qquad I_{\mathrm{A}} = 17kA$$

Final modulation

$$I(z) = C I_0 \left[1 + \rho_{\mathrm{f}} \cos(Ckz + \phi)\right]$$

High gain linear approximation:

$$\rho_{\mathrm{ind}} \gg \rho_{\mathrm{i}}\ , \qquad G = \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{i}}} \simeq \frac{\rho_{\mathrm{ind}}}{\rho_{\mathrm{i}}} \gg 1$$

Amplification mechanism is similar to that in high-gain klystron

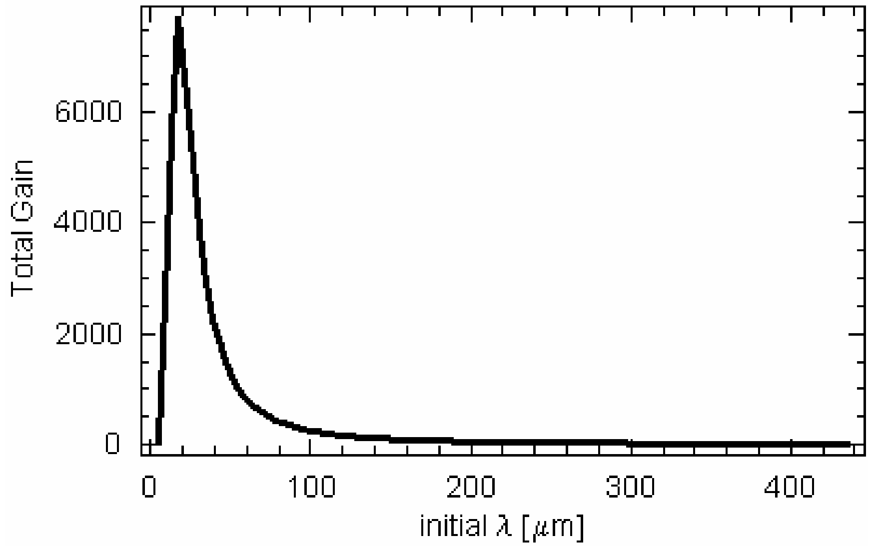
Gain formula

The gain depends neither on phase of $Z(k)$ nor on sign of R_{56} :

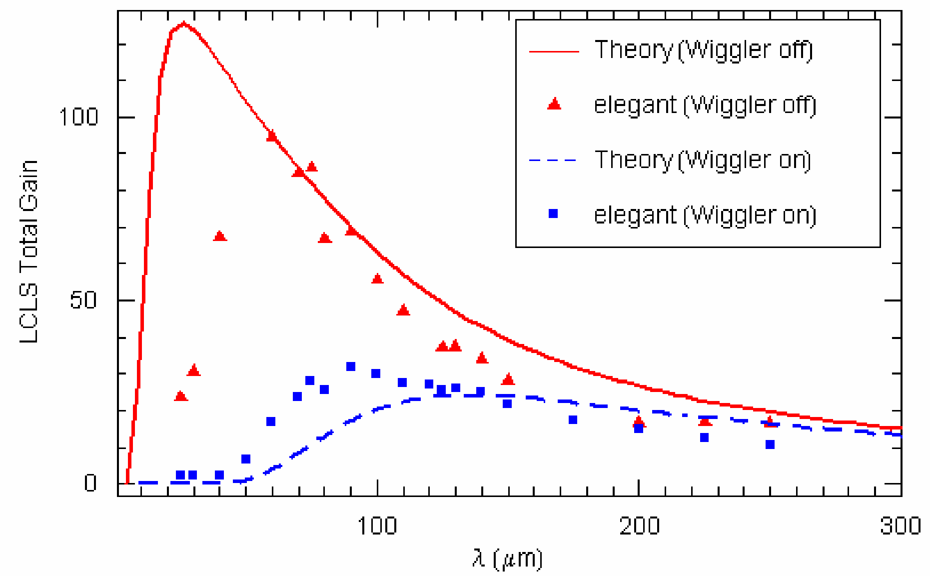
$$G = Ck|R_{56}|\frac{I_0}{\gamma_0 I_A}\frac{|Z(k)|}{Z_0}\exp\left(-\frac{1}{2}C^2k^2R_{56}^2\frac{\sigma_\gamma^2}{\gamma_0^2}\right)$$

LCLS Set-UP

- 3 keV intrinsic energy spread is too small for the high-frequency LSC

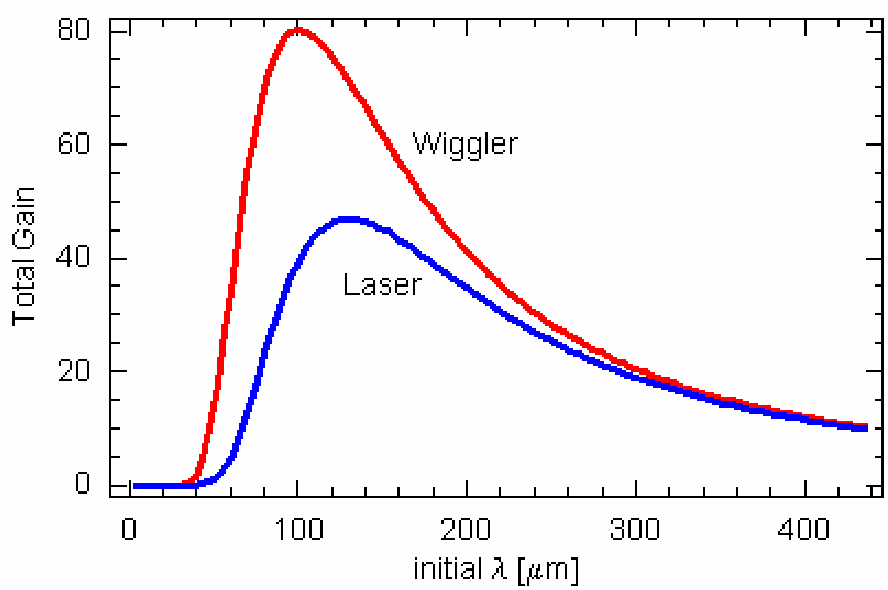


- Gain is high enough to amplify the shot noise bunching $b_{\text{eff}} \sim \frac{1}{\sqrt{N_\lambda}} \sim 10^{-4}$



Only CSR

Solution = Wiggler before BC2



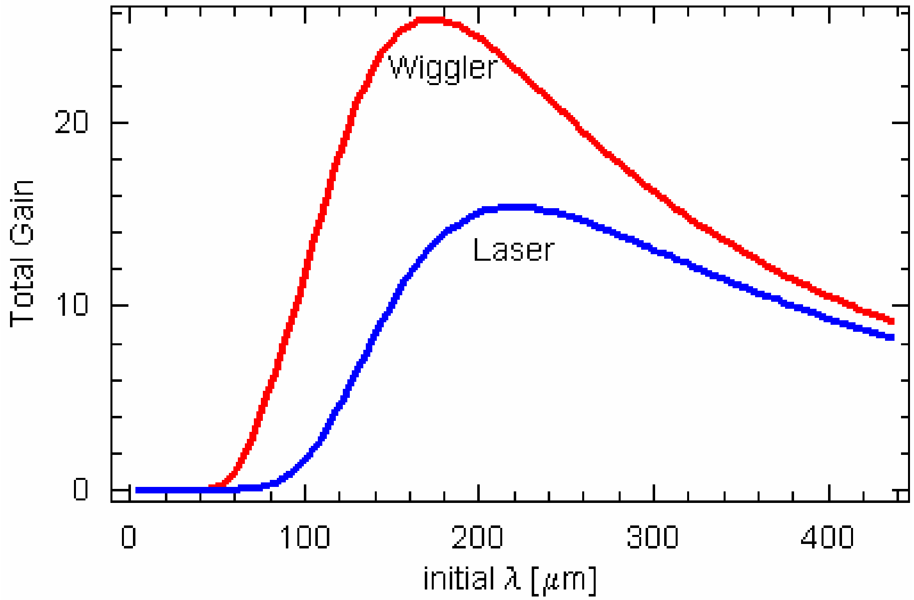
CSR + LSC

- **Peak gain increased by a factor of 3**
- **Solution = Adding Energy Spread on beam before BC1**

“laser heater”

3 keV-> 30 keV before BC1

Even More Landau Damping



• **Current design has energy spread 1x10-4 for the FEL**

• **Since FEL ρ ~ 5x10-4, small increase in energy spread is allowed, say 1.7x10-4**

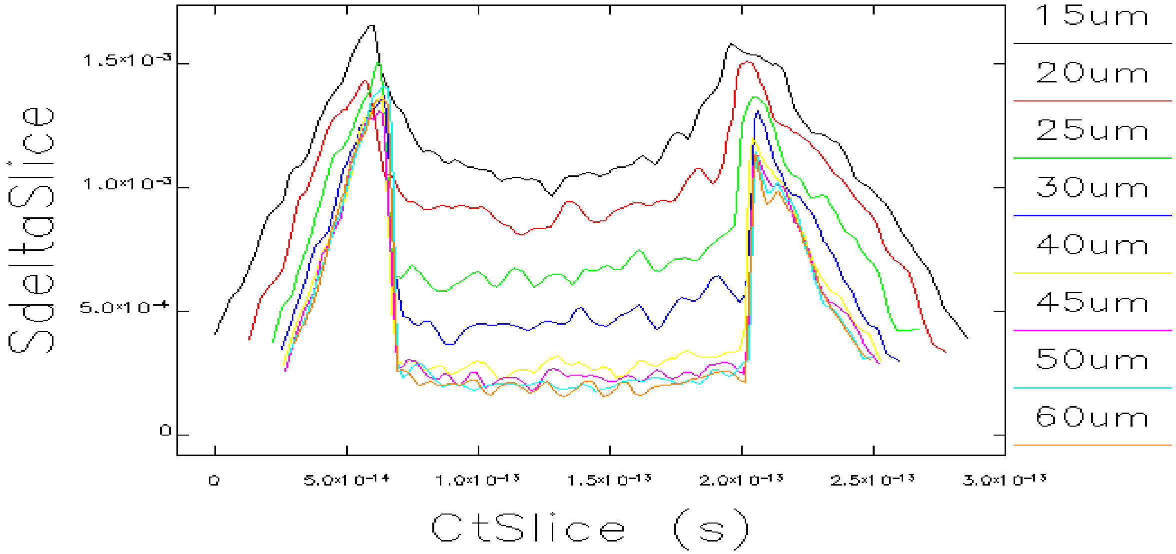
➔Laser heater increases energy spread 3 keV ➔ 50 keV

➔Wiggler increases energy spread to 5x10-5 at 4.5 GeV

Real danger is in fact energy spread > $\rho = 5.10^{-4}$

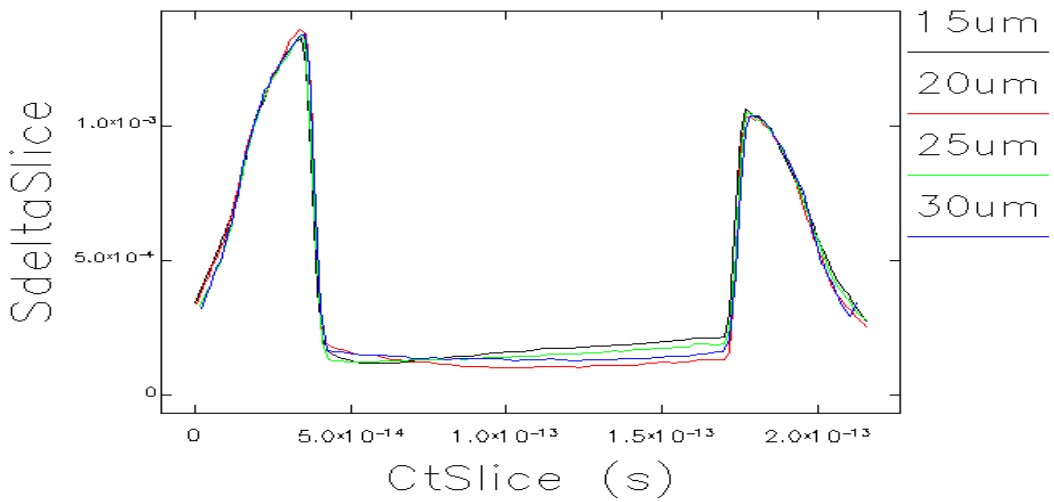
Wiggler

$\Delta\gamma$



Laser

$\Delta\gamma$



Elegant (from M.Borland)

- ***All the simulations are based on theory just described***
- ***Let's compare this theory with results from space charge simulation codes***

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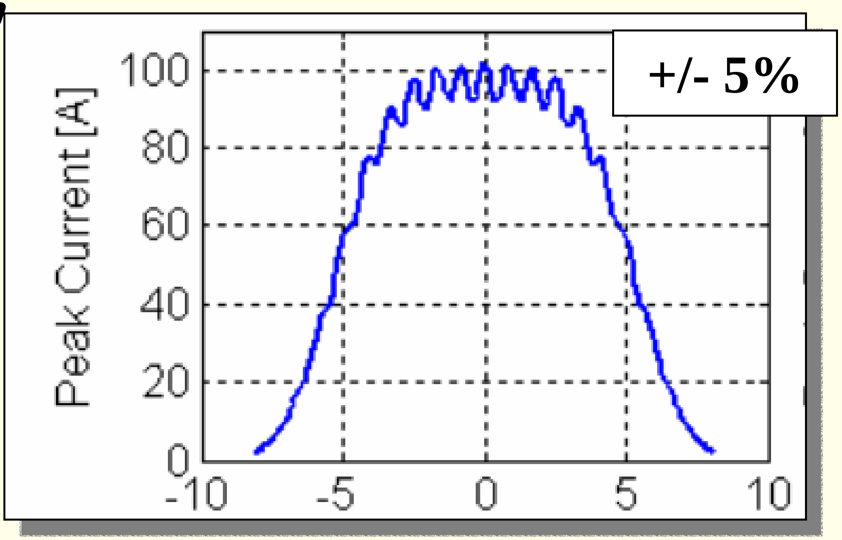
Simulations

- *Simulations description*
  - *40k/200k particles*
  - *Distribution generated using the Halton sequence of numbers*
  - *Longitudinal distribution*

$$\left[1 + A\cos(kz)\right] \; e^{-\left[(z-z_o)^4/\left(4\sigma^4\right)\right]}$$



  - *2.65 m of drift*
  - *With 3 cases studied*
    - *6MeV, 1nC*
    - *6 MeV , 2nC*
    - *12 MeV, 1nC*

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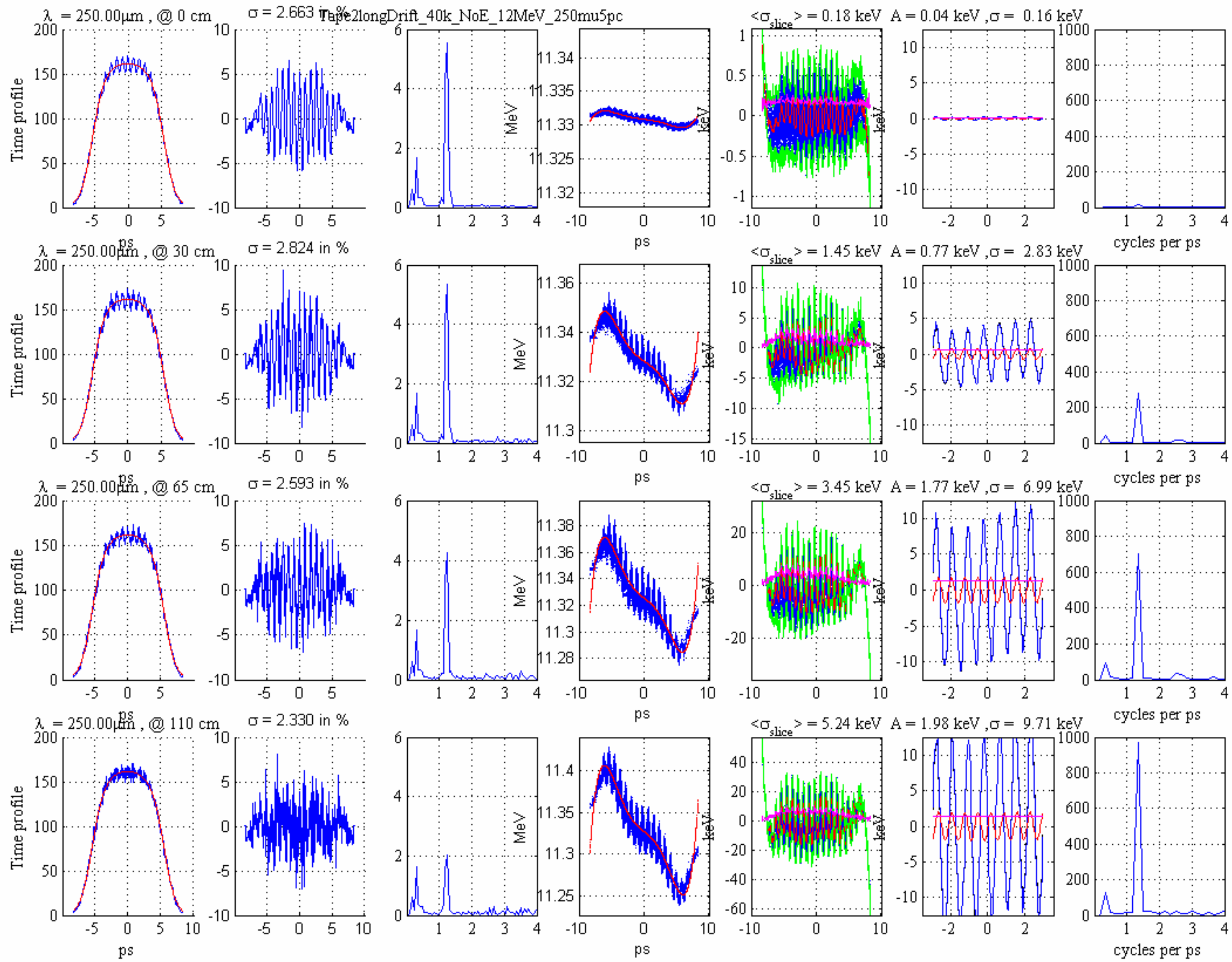

Theory Institute, ANL , 24 September 2003

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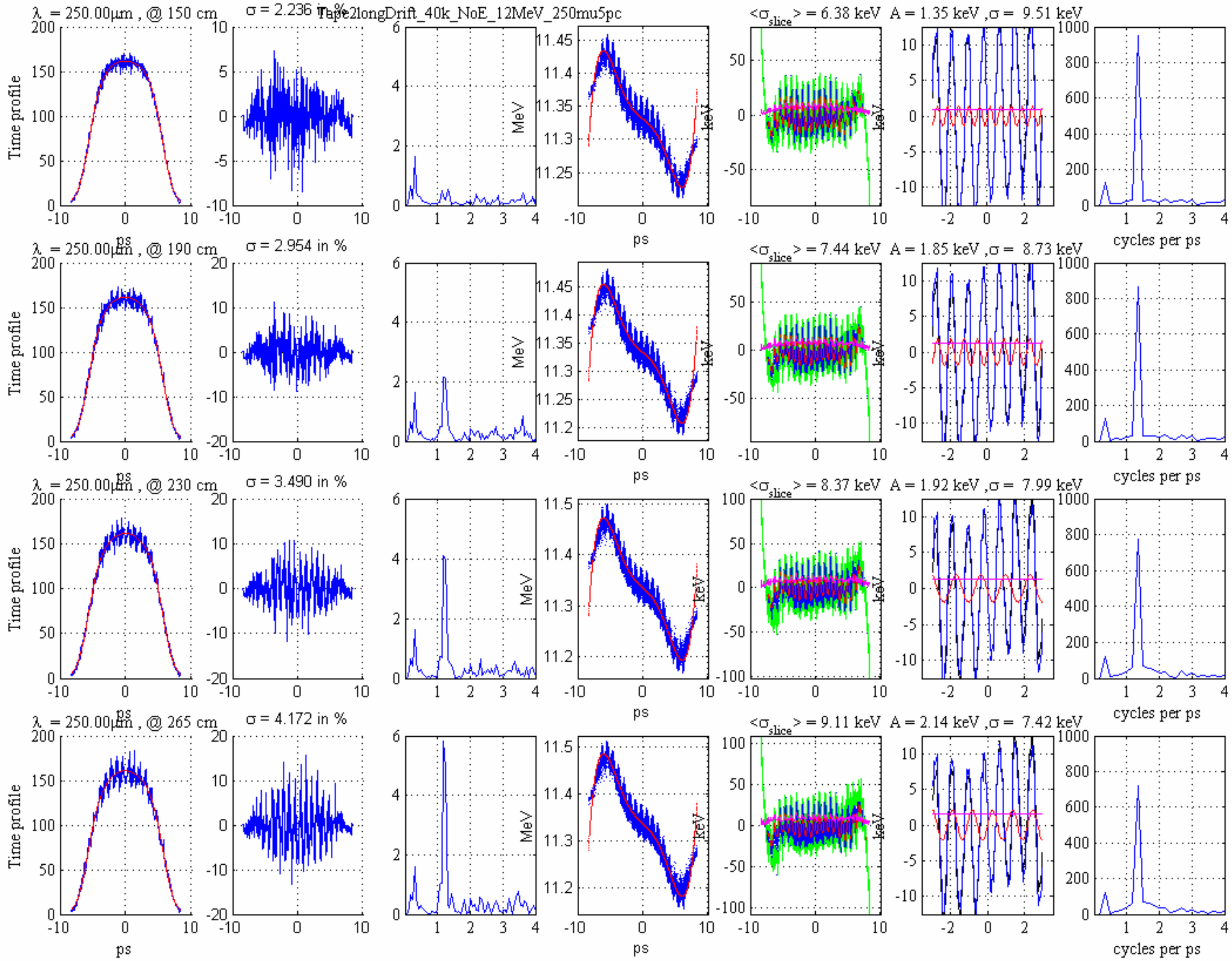

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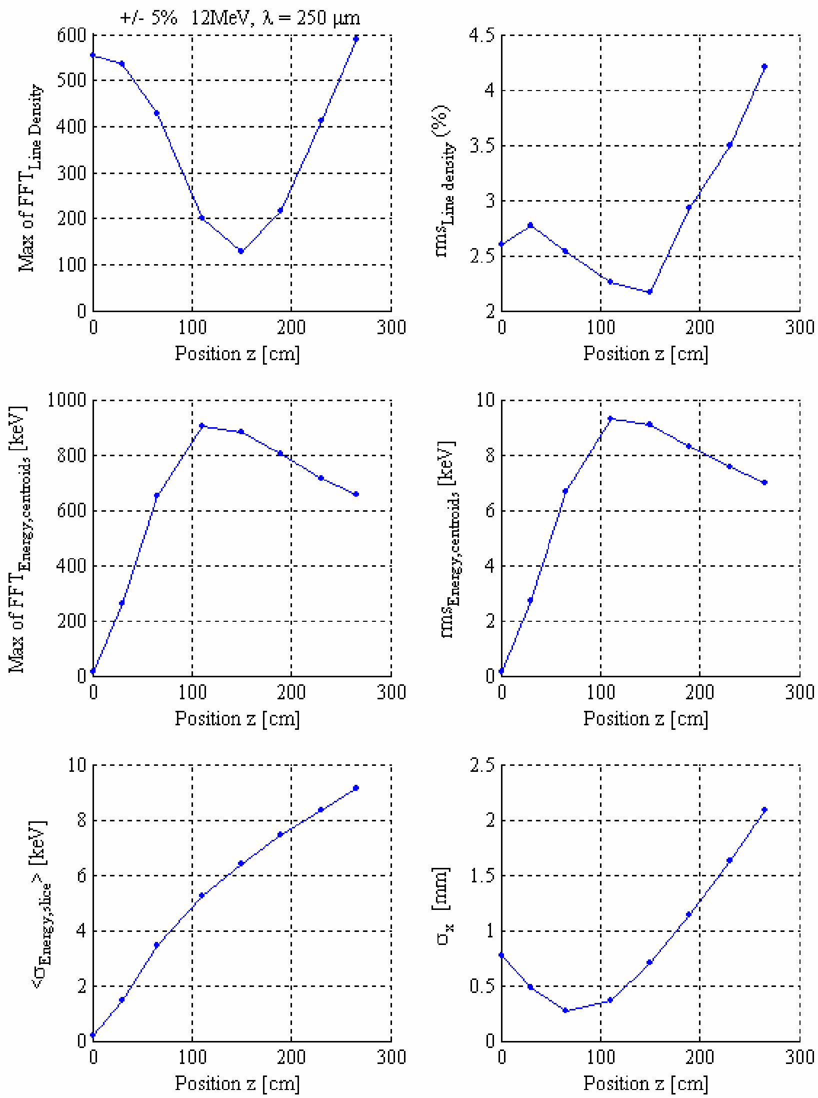
Half a plasma period, 12 MeV, 1nC, λ = 250 μm



Oscillation of density characterized by

" modulation amplitude"

current density ⇔ energy

But,

1- Residual energy modulation after half a period

2- Line density modulation amplitude strongly increased

Two parameters have also evolved along the beamline :

- uncorrelated energy spread

- transverse beam size

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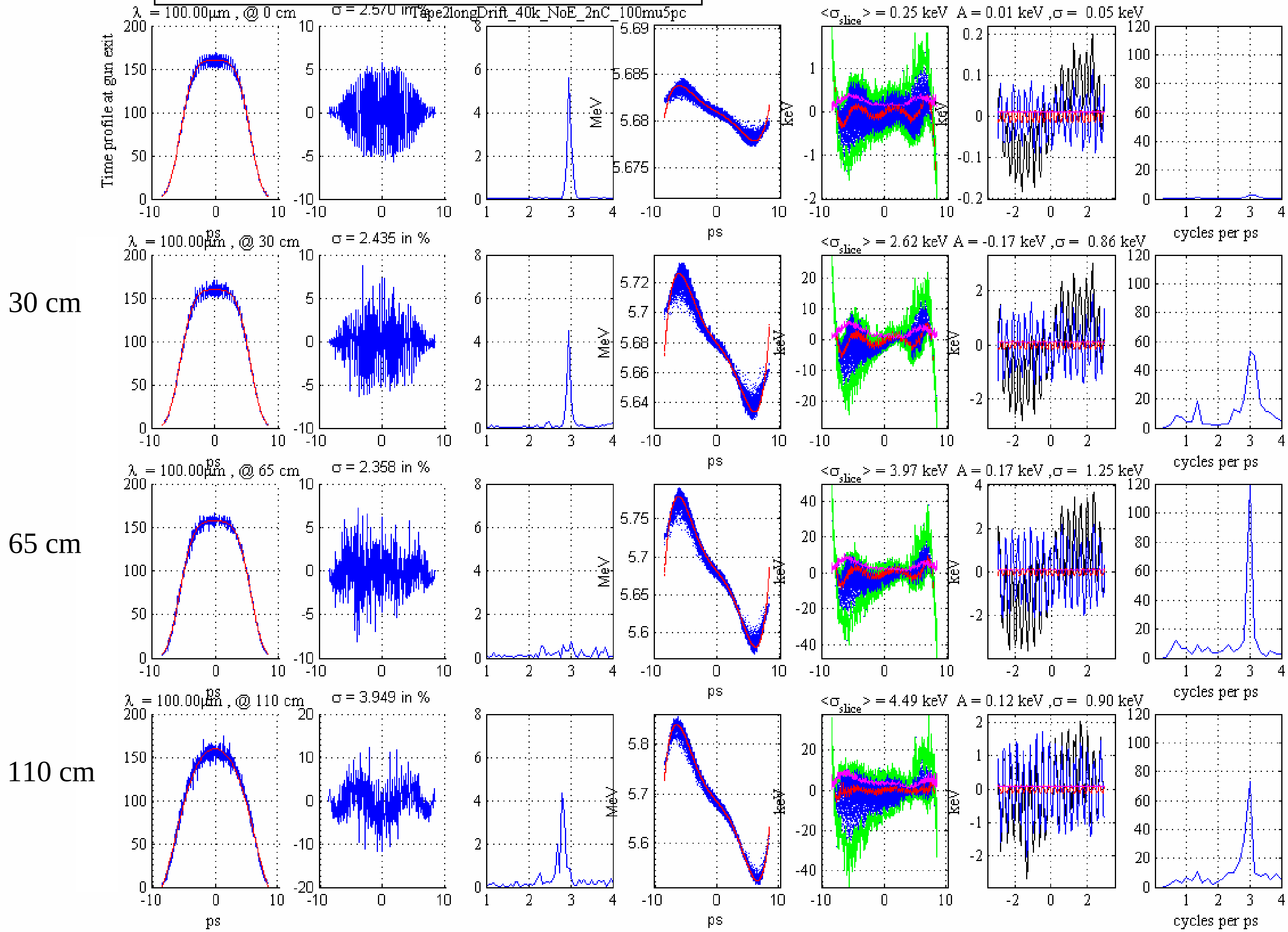
6 MeV, 2nC, λ = 100 μm



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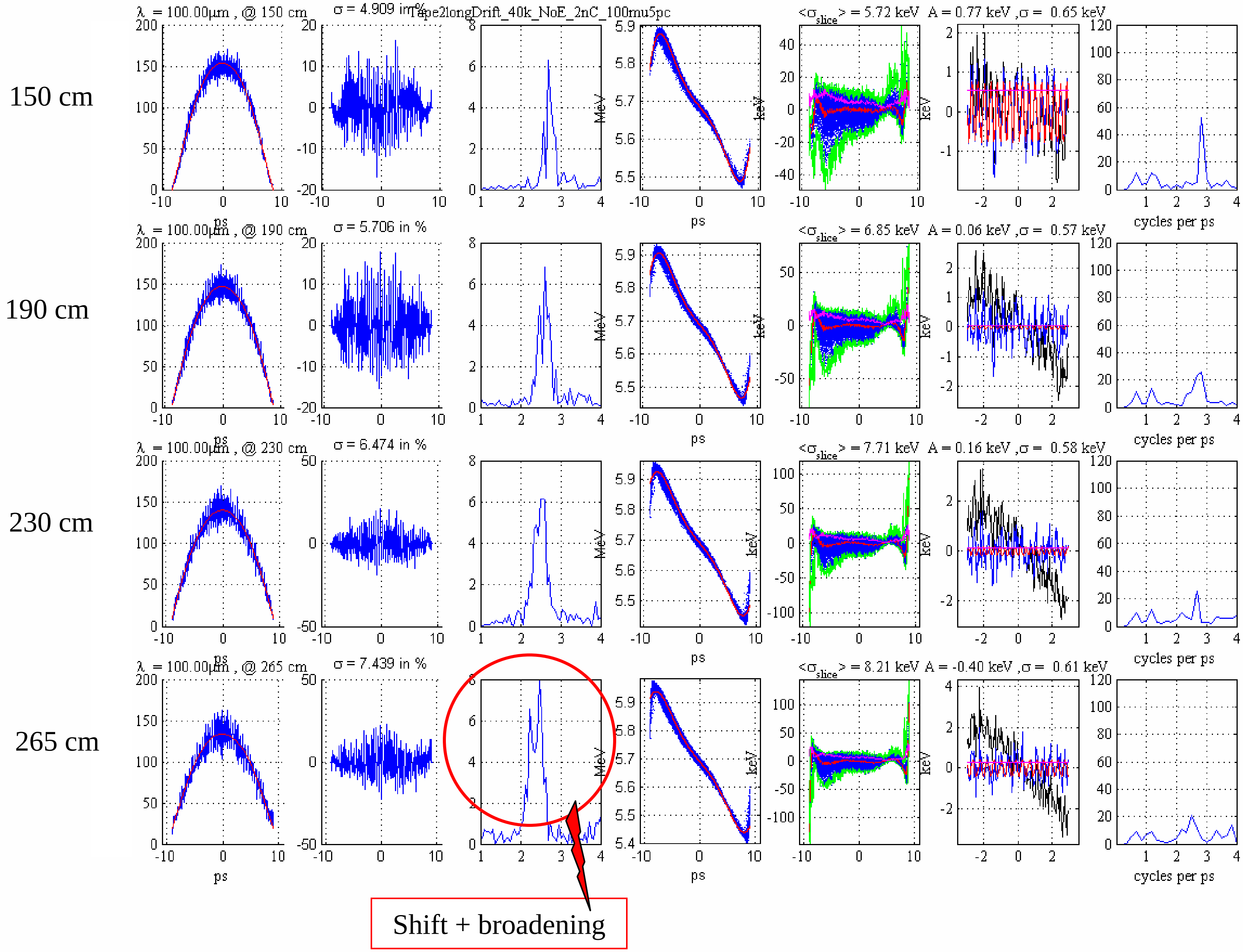

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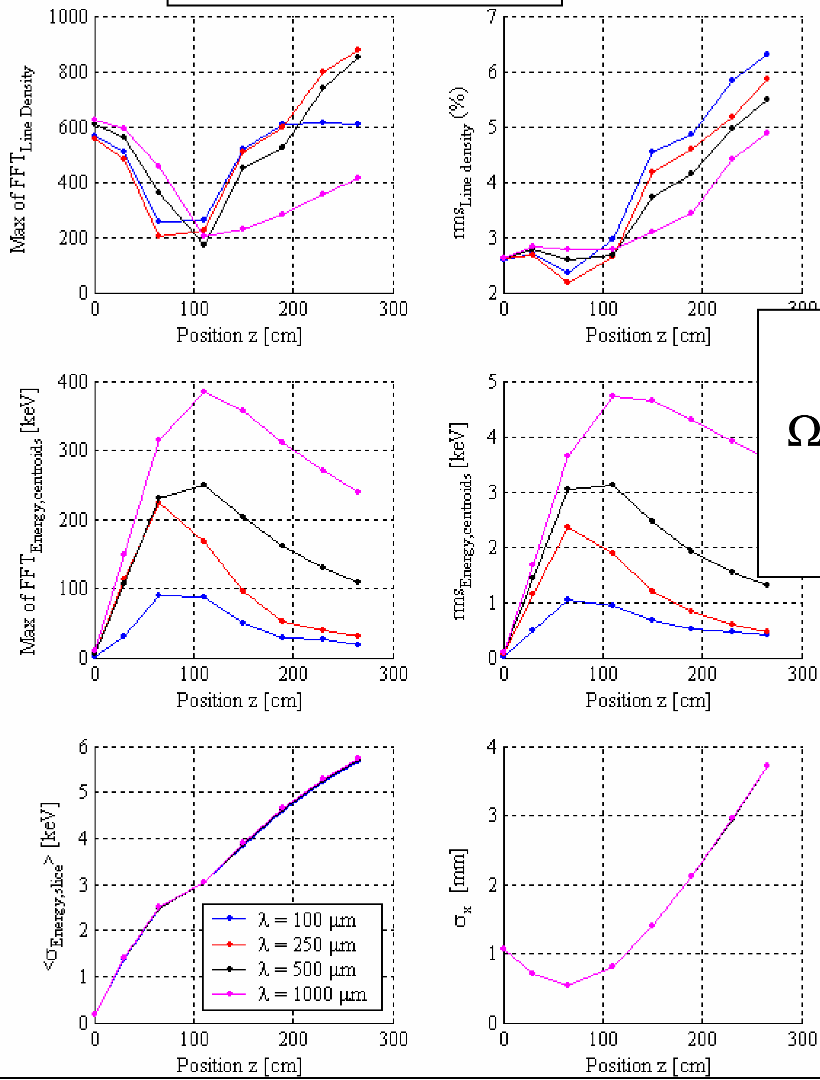
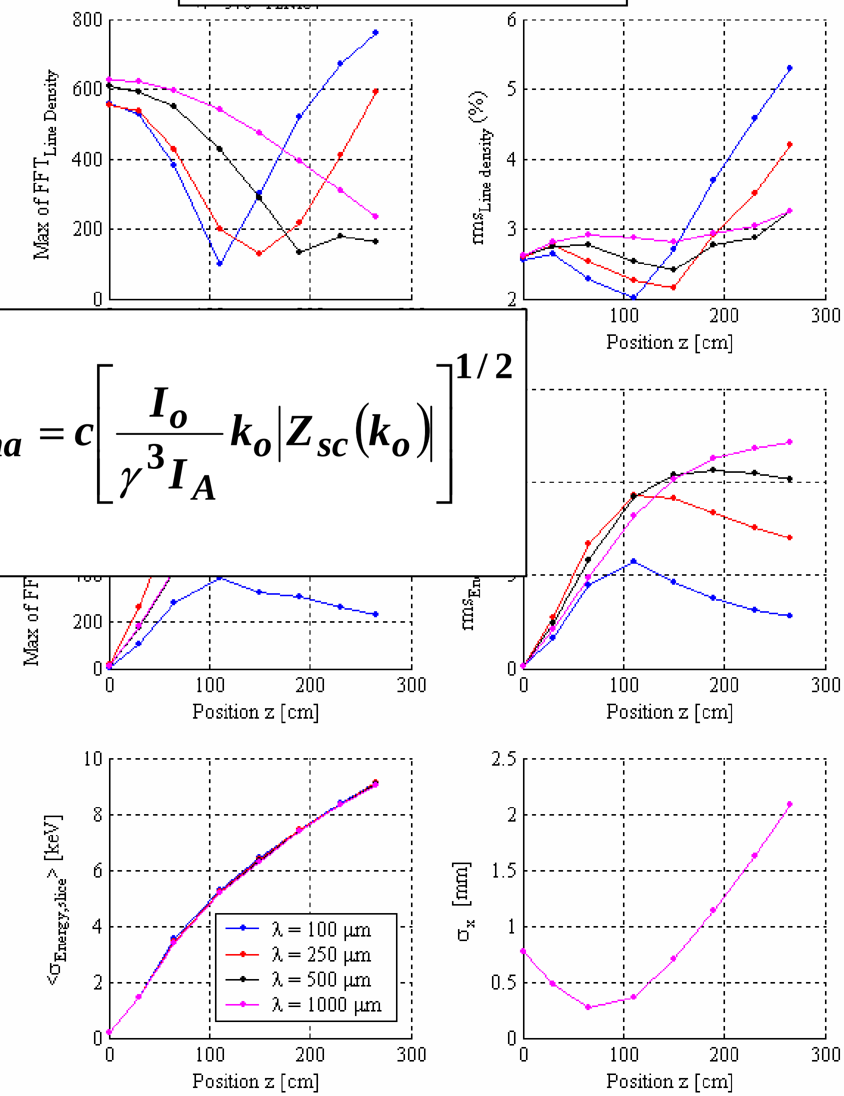
Evolution with energy

6 MeV , +/- 5%



12 MeV , +/- 5%



$$\Omega_{plasma} = c\left[\frac{I_o}{\gamma^3 I_A} k_o \left|Z_{sc}(k_o)\right|\right]^{1/2}$$

½ plasma period shorter for shorter wavelengths

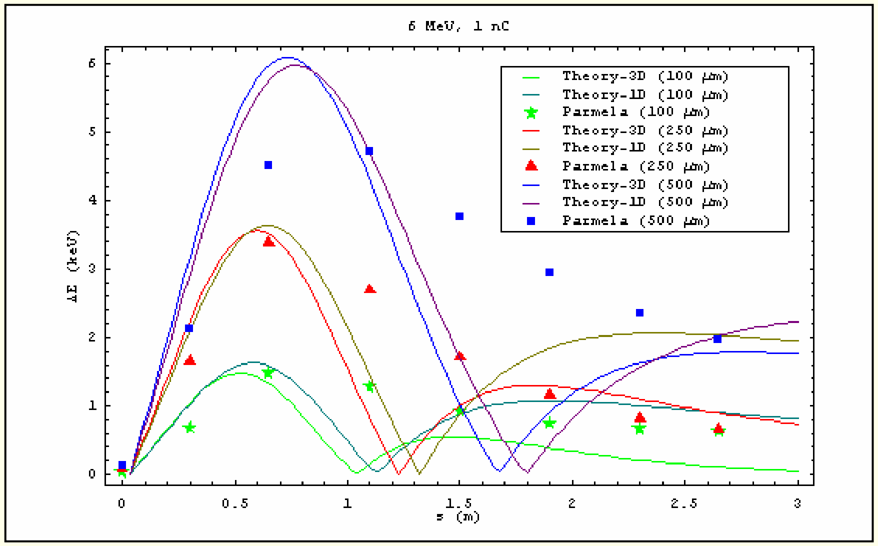
line density modulation ↑ with shorter wavelengths

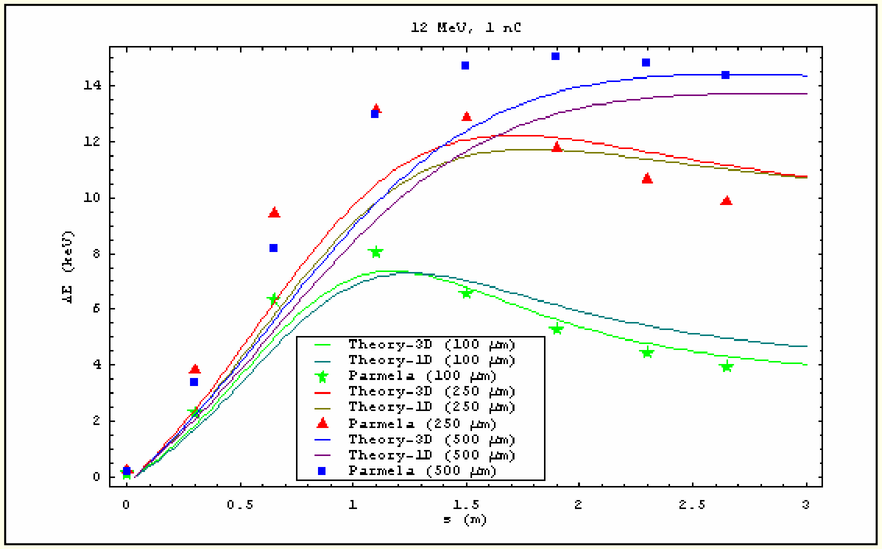
Energy modulation ↓ with shorter wavelengths

Amplitude Energy modulation gets larger than for 6 MeV

Uncorrelated energy spread gets larger by 60%

Comparison with theory





- Transverse beam size evolution along beamline taken into account

  (Radial variation of green's function for 2D)
- Evolution of peak current NOT taken into account yet
- Absence of dip in 6MeV curve :
  - "Coasting beam " against "bunched beam" with edge effects
  - Intrinsic energy spread

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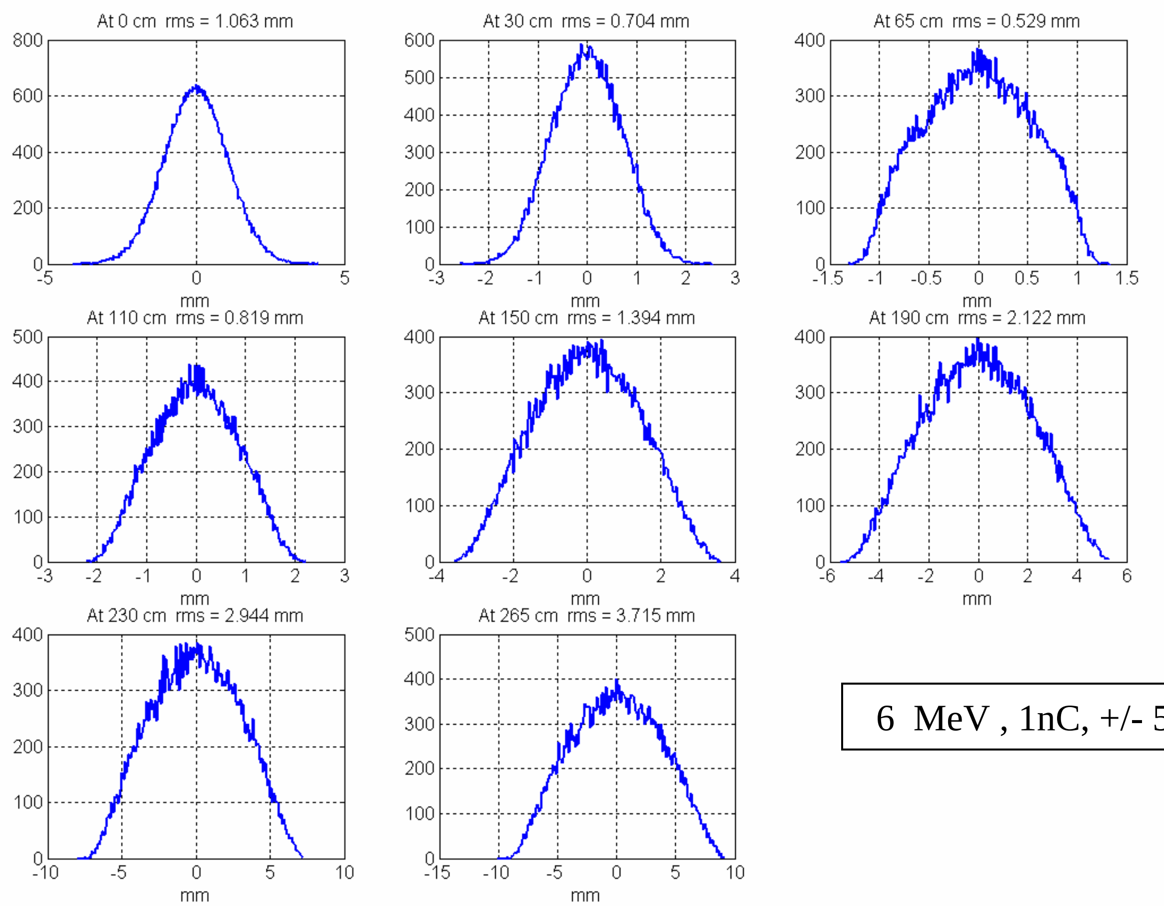
Evolution transverse profile



6 MeV , 1nC, +/- 5%

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3D model – impedance with r dependency

• 1-D model: transverse uniform pancake beam with longitudinal modulation

$$Z_{SC}(k) = \frac{4i}{kr_b^2}\left[1 - \frac{kr_b}{\gamma} K_1\left(\frac{kr_b}{\gamma}\right)\right]$$

where, r_b is the radius of the pancake beam.

• need to find realistic effective r_b

• radius dependence of impedance increases energy spread and so damping

• define $b(k;s,r)$ and $\Delta\gamma(k;s,r)$

• $$Z_{LSC}^{ring}(R,\alpha) = \frac{ik}{\gamma^2}\left[\Theta(\alpha - R)2K_0(\alpha)I_0(R) + \Theta(R-\alpha)\left\{\frac{2I_1(\alpha)}{\alpha} + \left[I_0(\alpha) + I_2(\alpha)\right]\right\}K_0(R)\right]$$

$$\alpha \equiv \frac{ka}{\gamma},\ \beta \equiv \frac{\sqrt{3}k\sigma_r}{\gamma} \text{ and } R \equiv \frac{kr}{\gamma}$$

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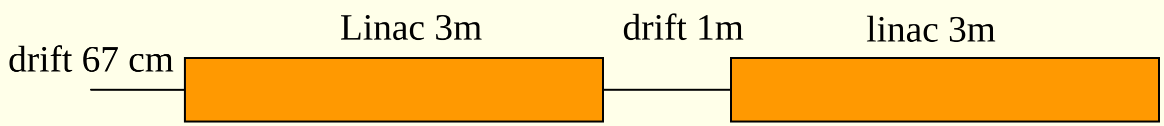
LSC in accelerating structures

Initial beam

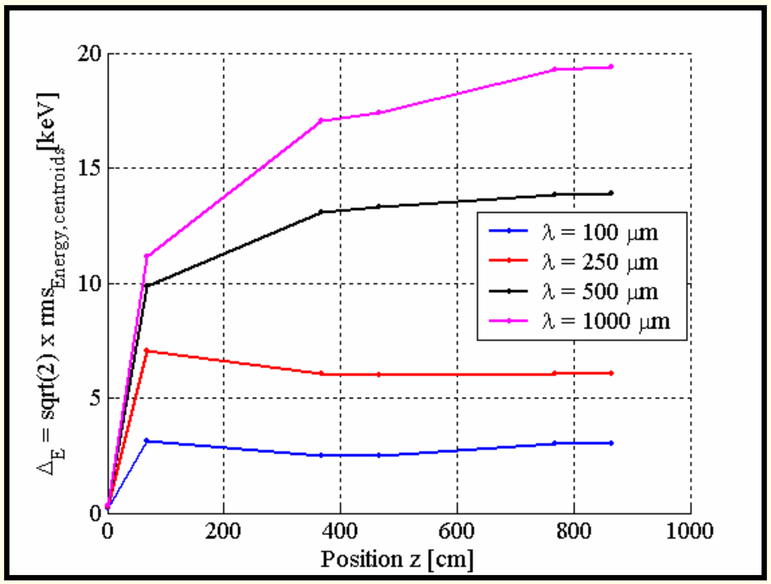
6MeV

No energy spread,

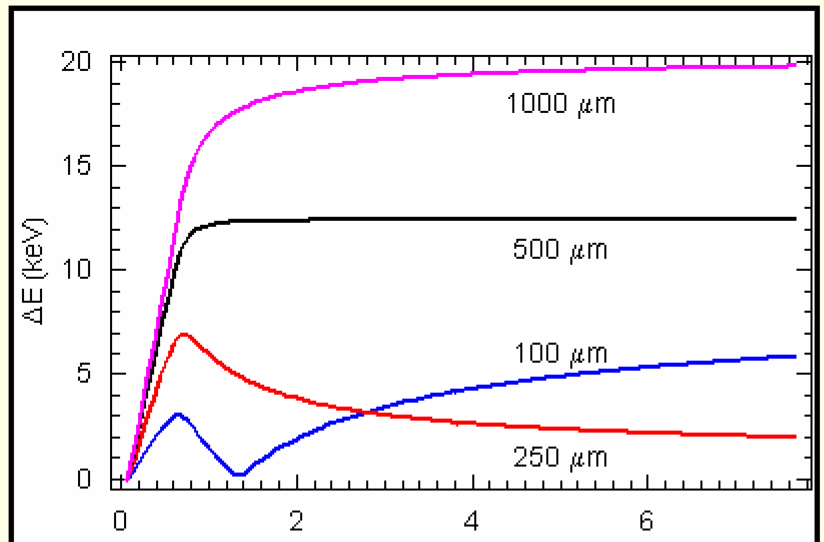
Current modulation M



Summary PARMELA simulations



Summary Theory [Huang,Wu]



Courtesy of Z.Huang

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- ***LSC in gun***
  - ***Modulation introduced on the laser (as in real life)***
  - ***Difficulties to study the effect for the LCLS pulse***
    - ***Study of*** λ = 50 μm , dz = 5 μm
    - ***Δz ~ 5 mm***
    - Nz x Nr ~ 1000 x 20
    - 200k particles
    - Per longitudinal bin ~ 200 particles , $1/\sqrt{200}$ ~7%

$\Rightarrow$ For initial modulation of less than 7% modulation of current density at gun exit is in noise

  - Case studied :
    - λ = 50 μm, 100 μm, 250 μm, 500 μm, 1000 μm
    - For +/-10% and +/- 20% initial modulation amplitude
  - Other cases under study
    - λ < 50 μm but for a shorter pulse than the LCLS pulse

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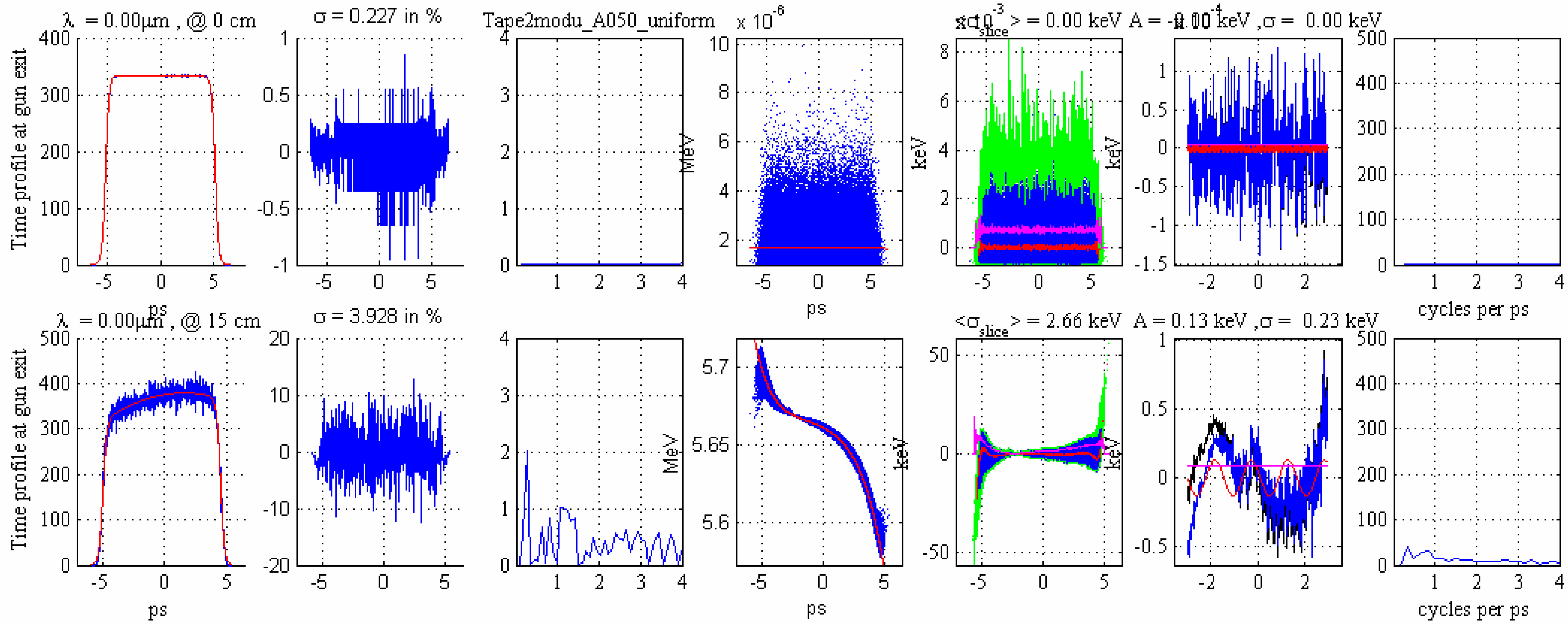
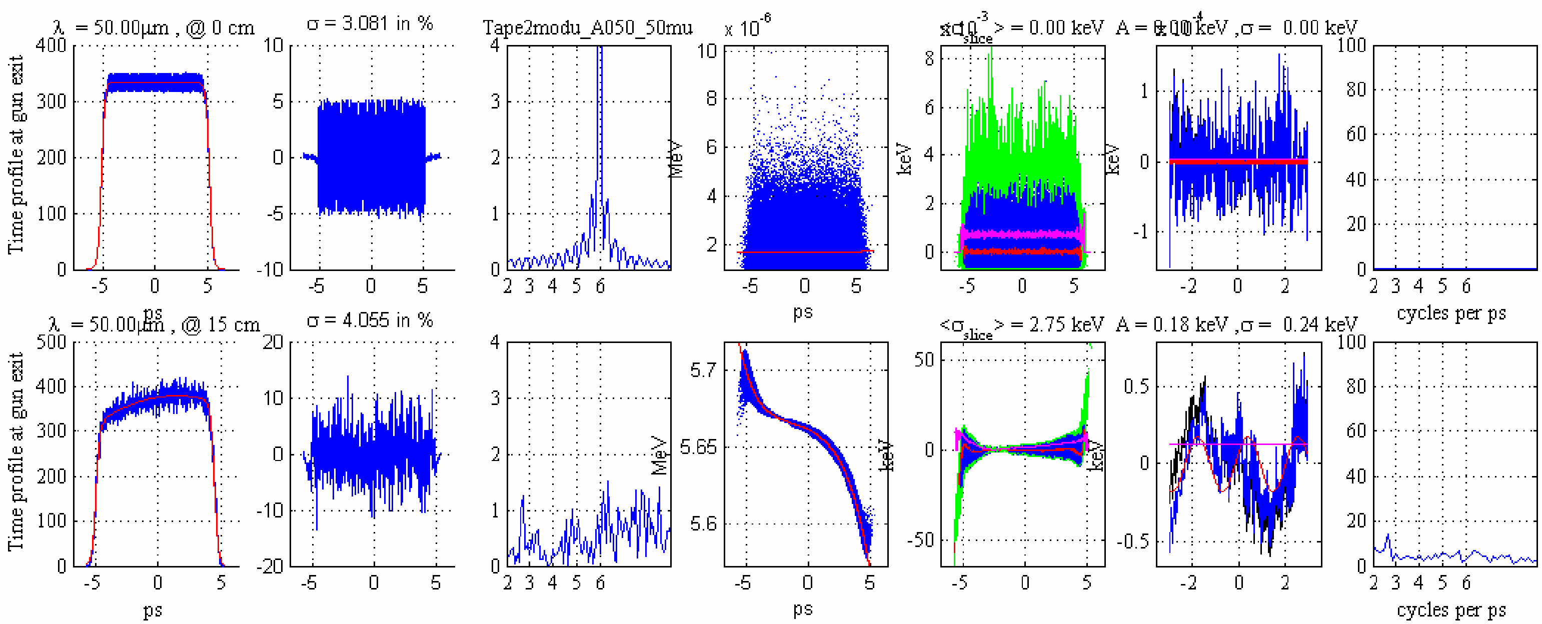
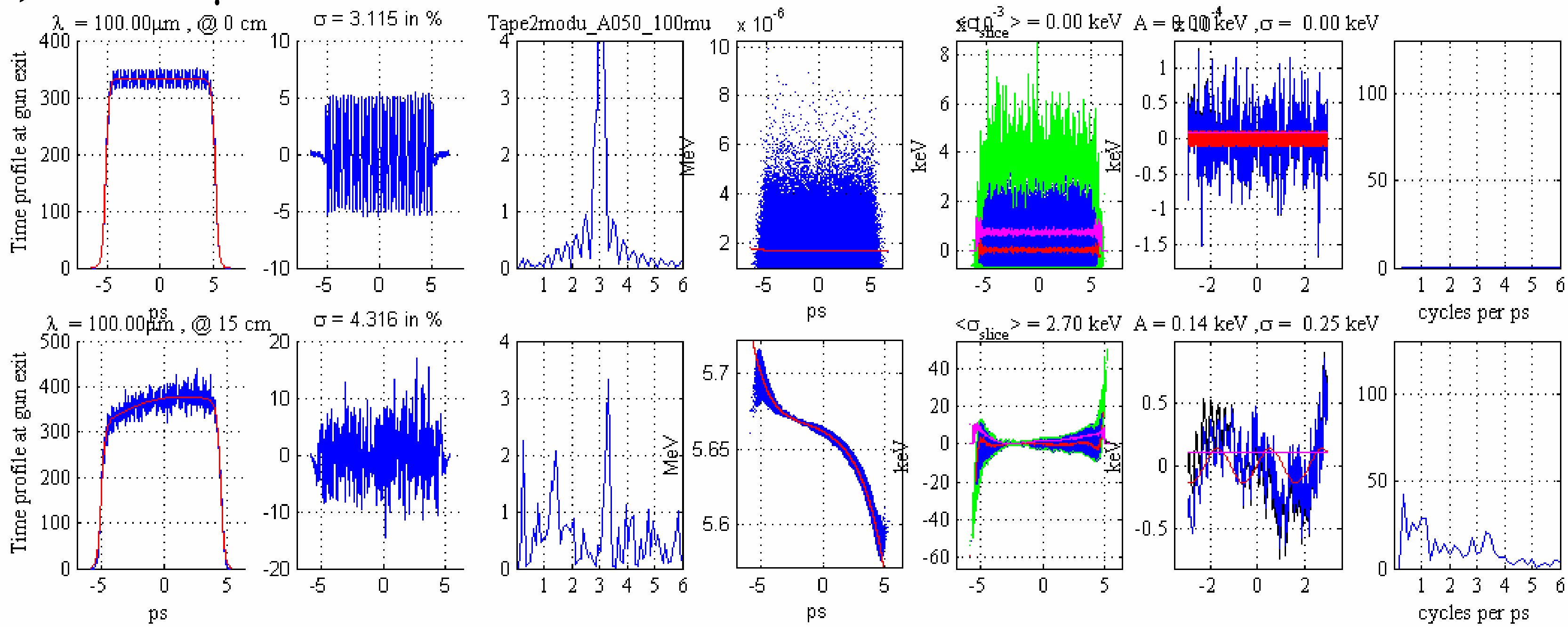

λ = 0.00µm , @ 0 cm
σ = 0.227 in %
Tape2modu_A050_uniform
Time profile at gun exit
MeV
keV
cycles per ps
ps
λ = 0.00µm , @ 15 cm
σ = 3.928 in %
<σslice> = 2.66 keV A = 0.13 keV ,σ = 0.23 keV

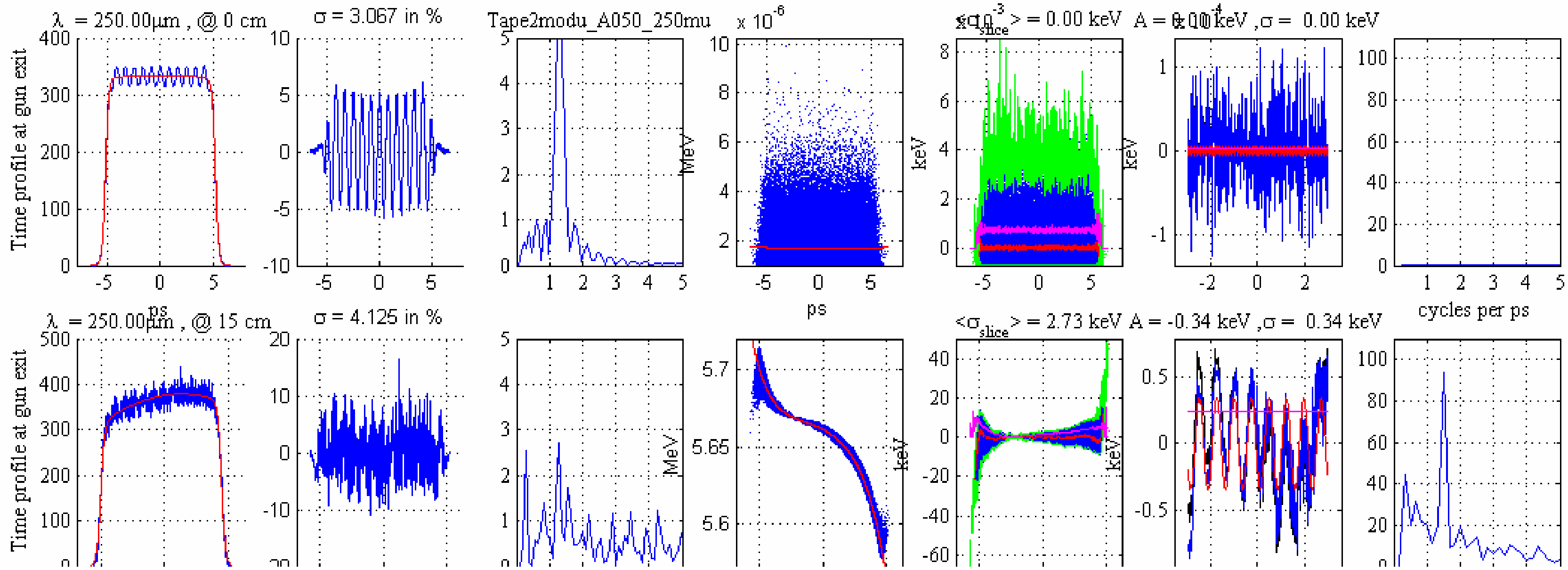
+/-5% , λ = 50 µm


λ = 50.00µm , @ 0 cm
σ = 3.081 in %
Tape2modu_A050_50mu
Time profile at gun exit
MeV
keV
cycles per ps
ps
λ = 50.00µm , @ 15 cm
σ = 4.055 in %
<σslice> = 2.75 keV A = 0.18 keV ,σ = 0.24 keV

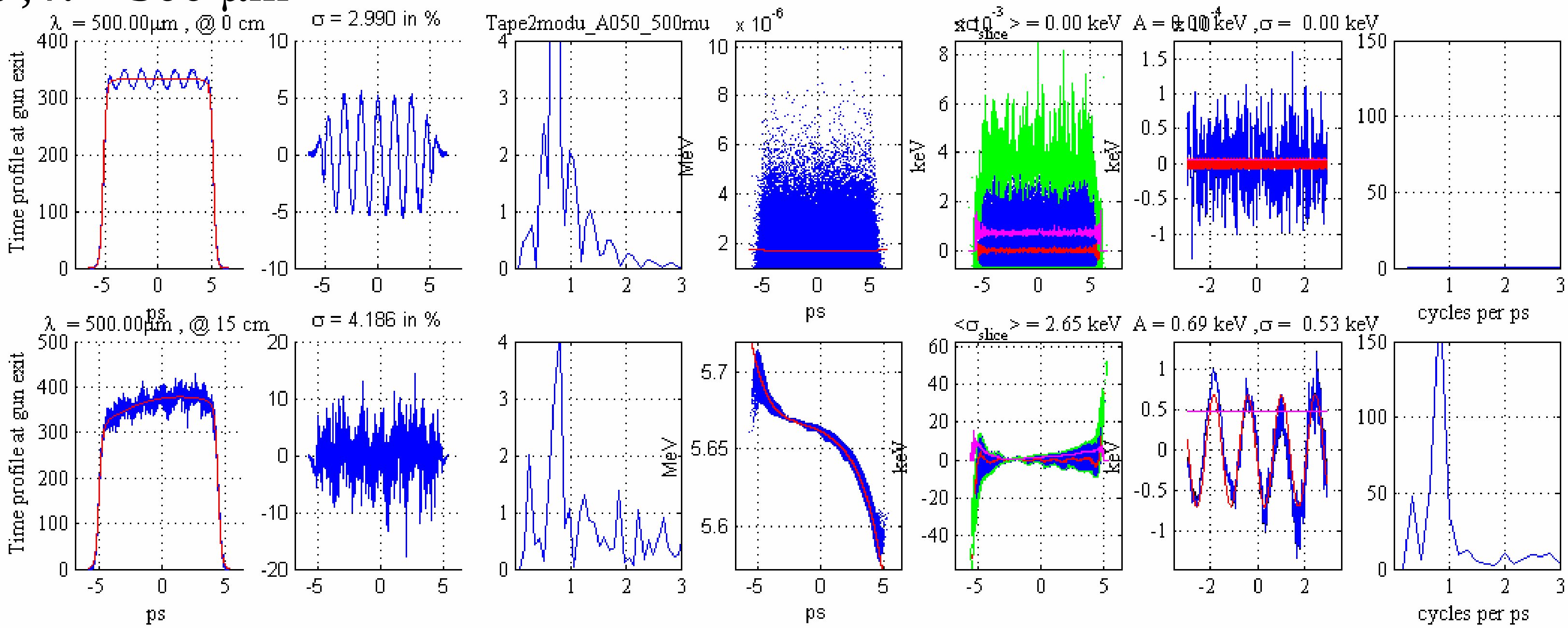
+/-5% , λ = 100 µm



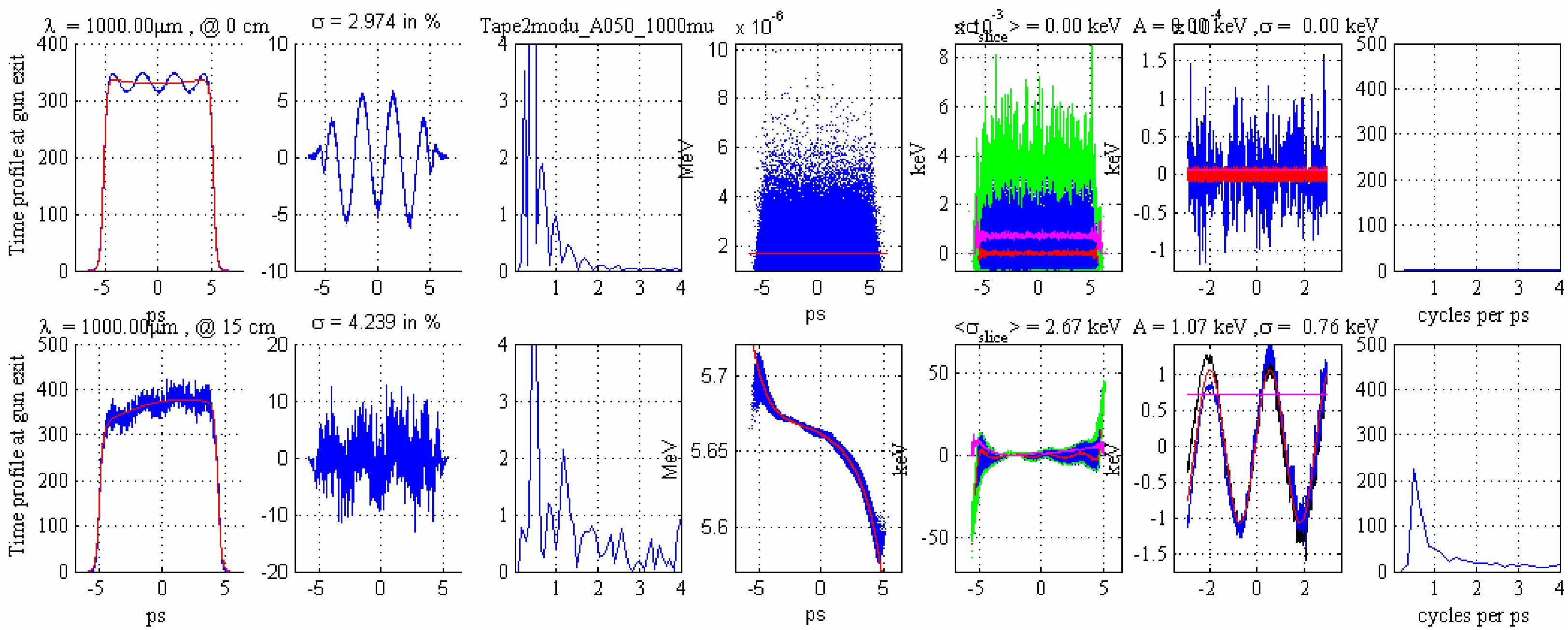
+/-5% , λ = 250 µm



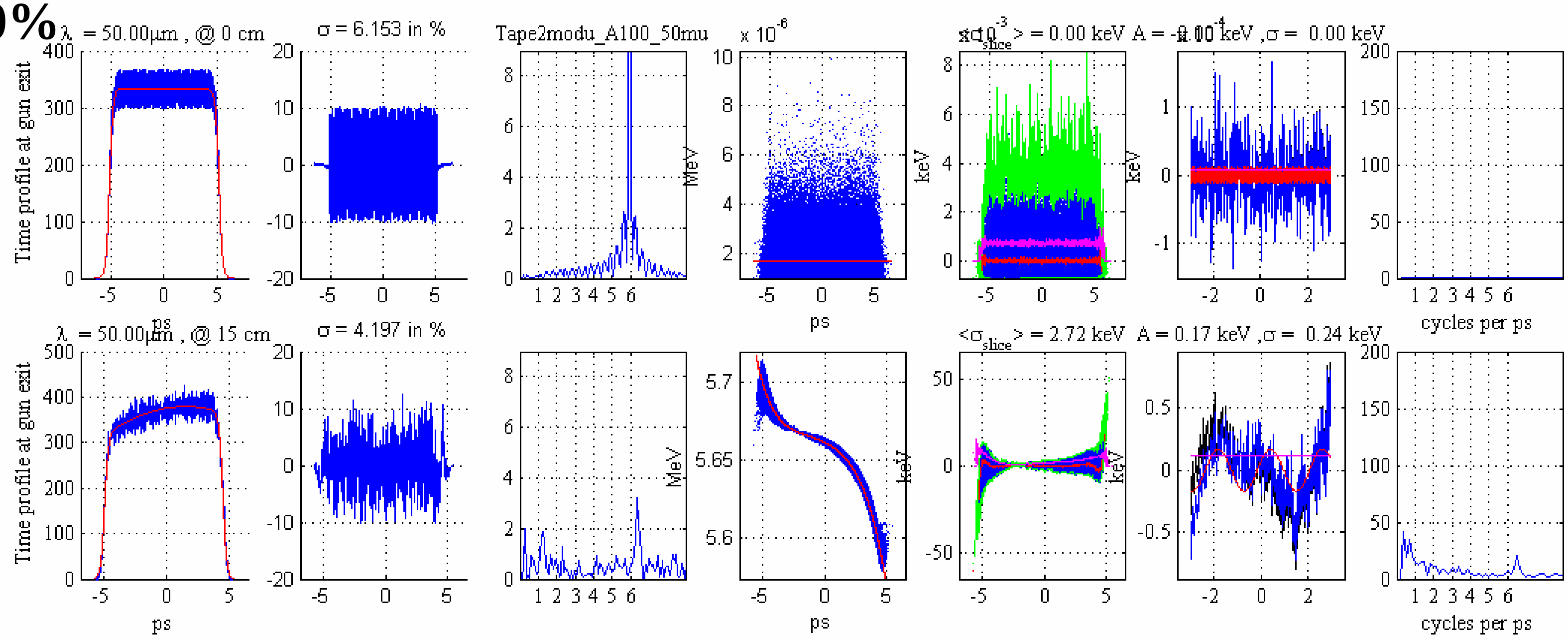
+/-5% , λ = 500 μm



+/-5% , λ = 1000 μm

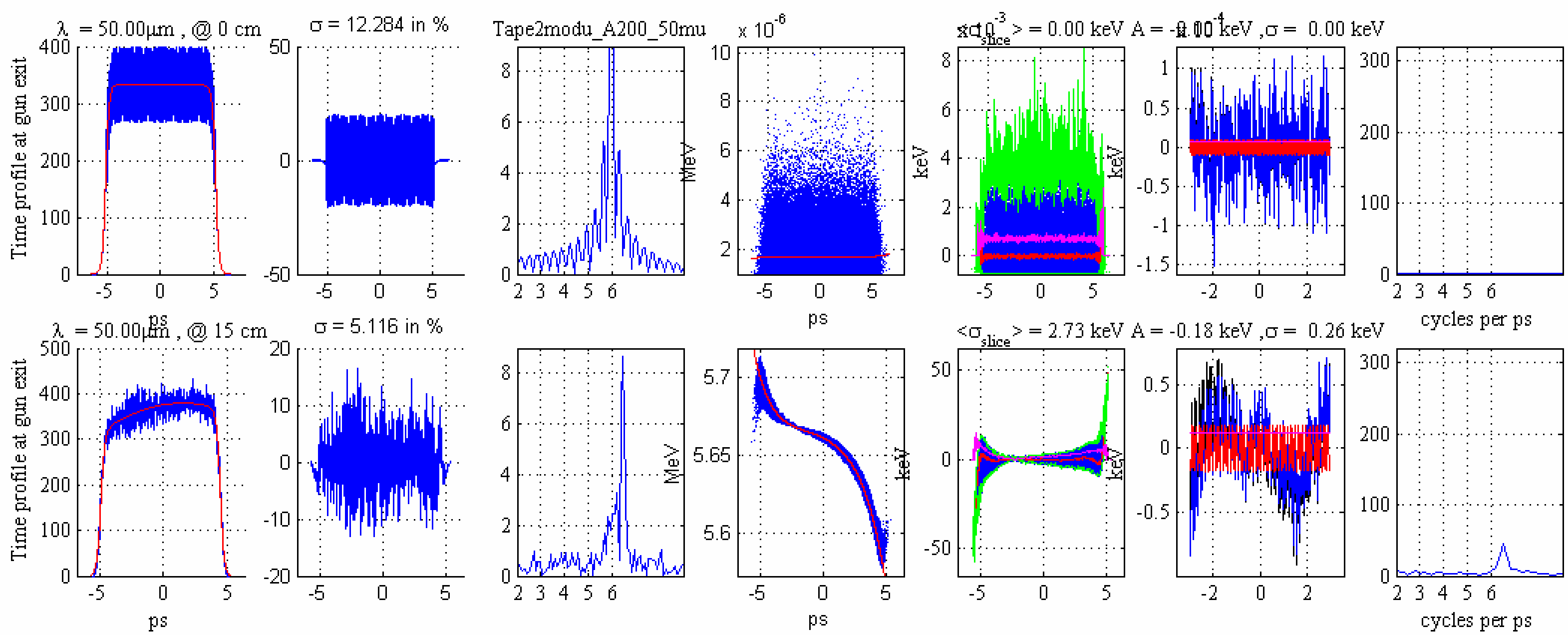


+/-10%

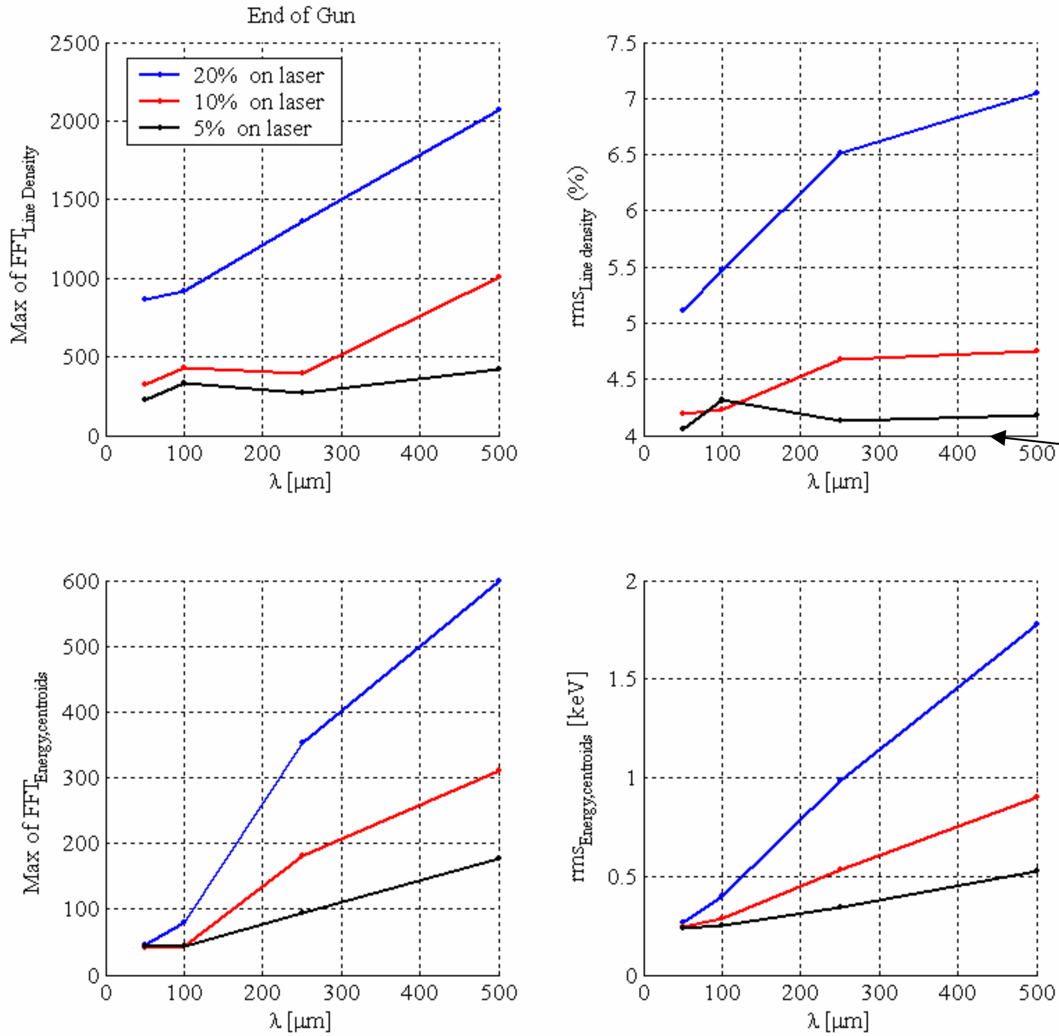


λ = 50.00μm , @ 0 cm
σ = 6.153 in %
Tape2modu_A100_50mu
Time profile at gun exit
ps
MeV
keV
cycles per ps
λ = 50.00μm , @ 15 cm
σ = 4.197 in %
<σslice> = 2.72 keV A = 0.17 keV ,σ = 0.24 keV

+/-20%



λ = 50.00μm , @ 0 cm
σ = 12.284 in %
Tape2modu_A200_50mu
Time profile at gun exit
ps
MeV
keV
cycles per ps
λ = 50.00μm , @ 15 cm
σ = 5.116 in %
<σslice> = 2.73 keV A = -0.18 keV ,σ = 0.26 keV



Noise Level = 4%

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LSC along the LCLS

- ***Modulation introduced on the laser (as in real life)***
- ***Difficulties to study the effect for the LCLS pulse***
  - ***Study of*** λ = 50 μm , dz = 5 μm
  - ***Δz ~ 5 mm***
  - Nz x Nr ~ 1000 x 20
  - 200k particles
  - Per longitudinal bin ~ 200 particles , 1/√200 ~7%

⇒ For initial modulation of less than 7% , modulation in noise

- Case studied :
  - λ = 50 μm, 100 μm, 250 μm, 500 μm, 1000 μm
  - For +/-10% and +/- 20% initial modulation amplitude
- Other cases under study
  - λ < 100 μm but for a shorter pulse than the LCLS pulse

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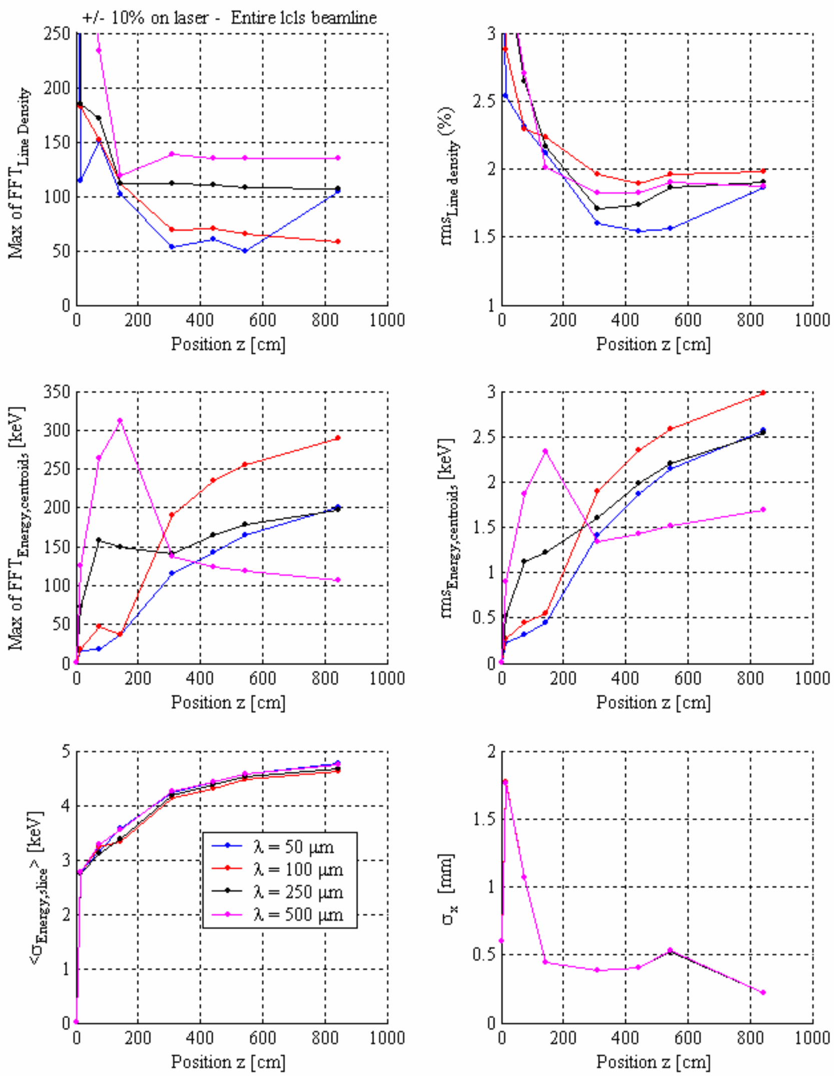
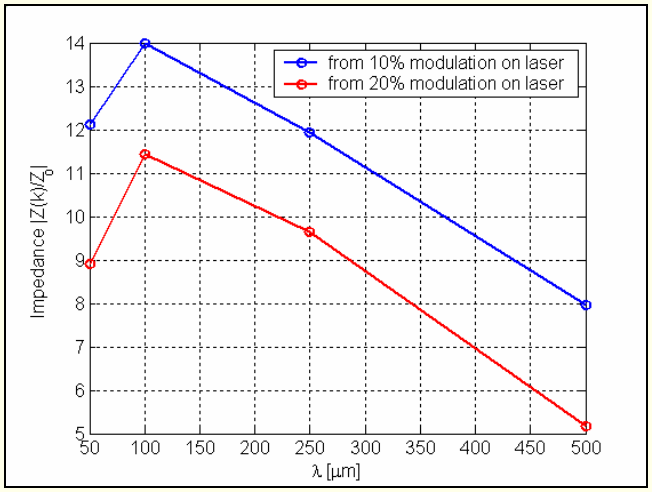


At end LCLS injector beamline:

- Current density modulation strongly attenuated
- residual energy oscillation has amplitude between 2 keV and 4 keV for wavelengths [50 μm, 500 μm]
- Impedance defined by

$$\Delta\gamma = \left|\frac{Z(k)}{Z_o}\right| \frac{I_o}{I_A} \rho_i$$

$$I(z) = I_o[1 + \rho_i \cos(kz)]$$



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Conclusion

- *Good agreement Simulations / Theory for drift and Acceleration*
- *Discrepancies are under investigation*
  - *Intrinsic energy spread increase*
  - *Edge effects for bunched beam*
- *More comparisons simulations /theory in high energy regime*
- *Difficulties to do simulations for small modulation amplitude*
  - *Noise Problem*
  - *Shorter wavelengths, not enough particles*
- *More comparisons experiments / simulations*
- *"Attenuation" in gun might make situation not as critical as first thought*
- *<u>But</u> not enough attenuation : extrapolation from 10% case*
  - *for wavelengths >100 μm : attenuation line density modulation by factor of 3*
  - *for wavelengths <100 μm : attenuation line density modulation by factor of 4*

- *<u>Beam Heater is under discussion for the LCLS beamline</u>*

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