LS-29
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Nonideal Undulator Spectra

The undulator spectra may have harmonic broadening due to the angular divergence and energy spread of the electron beam in the storage ring, variation of the undulator spatial period, and nonideal magnetic field distribution between the gap of the undulator. In most cases the energy spread of the electron beam seems to be rather small. The correction of the nonuniformity of the undulator period may be easier compared to that of the magnetic field distribution in an undulator.

This note calculates the undulator spectra under the following assumptions. The electron beam has divergences in the horizontal and vertical directions with an overall Gaussian distribution of the divergence. The undulator period is constant and magnetic field distribution is sinusoidal according to

$$\vec{B}_u = (B_x, B_y, B_z) = (0, B \sin\frac{2\pi z}{\lambda_u}, 0),$$

where the peak value B is <u>not</u> a constant. This means that the deflecton parameter K = 0.934 $B\lambda_u$ varies, but $\omega_o = 2\pi\beta^* c/\lambda_u$ is a constant.

Following is a summary of the calculation of an ideal N-unit undulator spectra (see LS-8):

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\ \vec{n}\times(\vec{n}\times\vec{\beta})\, e^{i\omega[t-\vec{n}\cdot\vec{r}(t)/c]}\right|^2$$

$$= \frac{4e^2\gamma^2}{c}\left|e^{iN\pi\nu} N\left(\frac{\sin N\pi\nu}{N\sin\pi\nu}\right) g_\nu(p,q)\right|^2$$

$$\equiv I_N^2, \tag{1}$$

where

$$g_\nu(p,q) = \left(\frac{\nu}{1+\frac{K^2}{2}+\gamma^2\theta^2}\right)(\hat{x}\,g_x + \hat{y}\,g_y), \tag{2}$$

$$g_x = (\gamma\theta)\cos\phi\left[L_\nu(p,q) - \frac{\sin N\pi\nu}{\sin\pi\nu}\right] + \frac{K}{2}\left[L_{\nu+1}(p,q) + L_{\nu-1}(p,q)\right],$$

$$g_y = (\gamma\theta)\sin\phi\left[L_\nu(p,q) - \frac{\sin N\pi\nu}{\sin\pi\nu}\right],$$

$$L_\nu(p,q) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(\nu\alpha + p\sin\alpha + q\sin 2\alpha)}\,d\alpha,$$

$$\nu = \frac{\omega}{\omega_1},$$

$$\omega_1 = \frac{2\gamma^2\omega_0}{1+\frac{K^2}{2}+\gamma^2\theta^2},$$

$$p = \frac{2\nu K(\gamma\theta)\cos\phi}{1+\frac{K^2}{2}+\gamma^2\theta^2},$$

$$q = \frac{\nu K^2/4}{1+\frac{K^2}{2}+\gamma^2\theta^2}.$$

The integral $L_\nu(p,q)$ is a double series of Bessel functions, and g_x and g_y are horizontally (perpendicular to the undulator magnetic field) and vertically polarized components of the radiation, respectively.

In deriving Eq. (1), it should be noted that the phase factor $e^{iN\pi\nu}$ depends on the location of the orgin:

$$\int_0^{N\left(\frac{2\pi}{\omega_0}\right)} e^{i(\nu\omega_0 t - p\sin\omega_0 t + q\sin 2\omega_0 t)}\,dt$$

$$= \frac{1}{\omega_0}\int_0^{2\pi N} e^{i(\nu\alpha - p\sin\alpha + q\sin 2\alpha)}\,d\alpha$$

$$= \frac{2\pi}{\omega_0}\, e^{iN\pi\nu}\,\frac{\sin N\pi\nu}{\sin\pi\nu}\,L_\nu(p,q)\,.$$

Generally the phase factor for an N_2 undulator of Fig. 1 could be seen from the following:

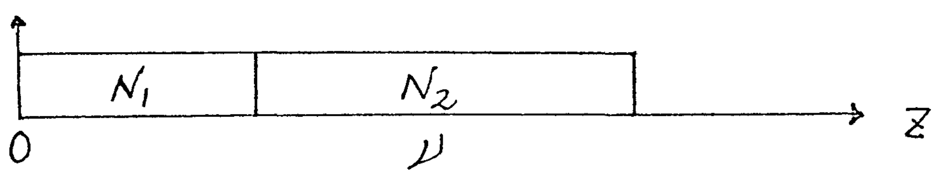

Fig. 1.

$$\int_{2\pi N_1}^{2\pi N_2} e^{i(\nu\alpha - p\sin\alpha + q\sin 2\alpha)}\,d\alpha$$

$$= e^{i2\pi N_1\nu}\, e^{iN_2\pi\nu}\,\frac{\sin N_2\pi\nu}{\sin\pi\nu}\,L_\nu(p,q)\,. \tag{3}$$

In Eq. (3), N_1 is the number undulator periods between the origin and N_2 system, and ν is the harmonic of the N_2 system (not N_1 system).

Consider an undulator system of Fig. 2. (N_i and N_j system)

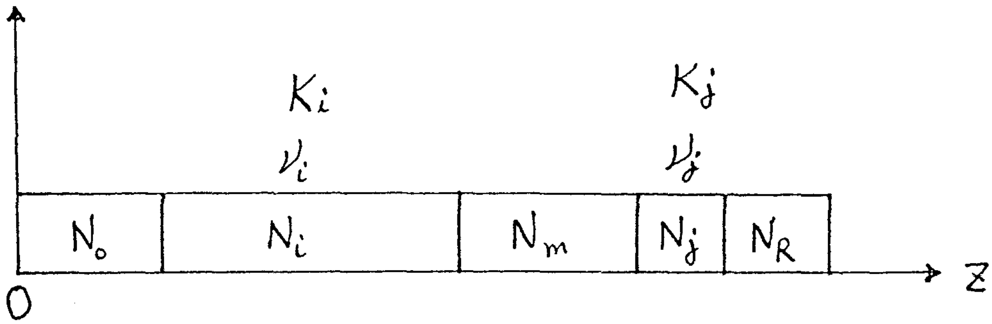

Fig. 2

The spectra may be expressed as

$$\frac{4e^2\gamma^2}{c}\left| e^{i2\pi N_0 \nu_i} e^{i\pi N_i \nu_i} \frac{\sin N_i \pi \nu_i}{\sin \pi \nu_i} g_{\nu_i}(p_i, q_i) \right.$$

$$\left. + e^{i2\pi(N_0 + N_i + N_m)\nu_j} e^{i\pi N_j \nu_j} \frac{\sin N_j \pi \nu_j}{\sin \pi \nu_j} g_{\nu_j}(p_j, q_j) \right|^2$$

$$= I_{N_i}^2 + I_{N_j}^2 + \left(I_{N_i} \cdot I_{N_j}^* + c.c \right). \qquad (4)$$

For a case of $N_{total} = \sum_{i=1}^{n} N_i$, Eq. (4) could be generalized as

$$\frac{d^2I}{d\omega\, d\Omega} = \sum_{i=1}^{n} I_{N_i}^{2} + 2\sum_{i<j}^{n} \mathrm{Re}\left\{ I_{N_i} \cdot I_{N_j}^{*} \right\}.$$

The interference term in Eq. (4) becomes

$$\frac{4e^2\gamma^2}{c}\, 2\cos\pi\left(n_i \nu_i + n_{ij}\Delta\nu_{ij} \right) \cdot$$

$$\cdot \left(\frac{\sin N_i \pi \nu_i}{\sin \pi \nu_i} \right) \left(\frac{\sin N_j \pi \nu_j}{\sin \pi \nu_j} \right) g_i\, g_j\,, \qquad (5)$$

where

$$n_i = N_i + 2N_m + N_j,$$

$$n_{ij} = 2N_o + 2N_i + 2N_m + N_j,$$

$$\Delta\nu_{ij} = \nu_j - \nu_i,$$

$$g_i = g_{\nu_i}(\rho_i, \xi_i).$$

The apparent dependence of the phase factor $e^{i2\pi N_o \Delta\nu_{ij}}$ on the orgin of the coordinate system is due to the variation of the observation angle θ.

Now consider a simple case of $N_j = 1$ and $\Delta\nu_{ij} = \Delta\nu$. Since the function $g_i g_j$ is a rather slowly varying function, nonvanishing values of Eq. (5) exist near the integer values of ν_i within a given opening angle of the radiated beam. A detailed calculation of the brightness of the interference term, Eq.

(5), at an angle ϕ, for a Guassian distribution becomes

$$\frac{4e^2\gamma^2}{\pi c\,\sigma_x'\sigma_y'}\Big[C\cos\pi(N_L+1)\Delta\nu + S\sin\pi(N_L+1)\Delta\nu\Big], \qquad (6)$$

where

$$C = W_c(\nu_0, A)\,g_i(I)\cdot g_j(I) + \sum_{J=I+1}^{J_{max}} e^{-(J-\nu_0)A}\, W_c(J,A)\,g_i(J)\cdot g_j(J),$$

$$S = W_s(\nu_0, A)\,g_i(I)\cdot g_j(I) + \sum_{J=I+1}^{J_{max}} e^{-(J-\nu_0)A}\, W_s(J,A)\,g_i(J)\cdot g_j(J),$$

$$N_L = 2N_0 + 2N_i + 2N_m + 1,$$

$$W_c(\nu_0, A) = \sum_{n=1}^{N_i} \frac{1}{A^2+[2\pi(N_m+n)]^2}\Big[A\cos 2(N_m+n)\pi\nu_0 - 2\pi(N_m+n)\sin 2(N_m+n)\pi\nu_0 - (-1)^{N_m+n} A\, e^{-(I+0.5-\nu_0)A}\Big],$$

$$W_s(\nu_0, A) = \sum_{n=1}^{N_i} \frac{1}{A^2 + [2\pi(N_m+n)]^2} \Big[A \sin 2(N_m+n)\pi\nu_0$$

$$+ 2\pi(N_m+n)\cos 2(N_m+n)\pi\nu_0$$
$$- (-1)^{N_m+n}\, 2\pi(N_m+n)\, e^{-(I+0.5-\nu_0)A}\Big],$$

$$A = \frac{1+K^2/2}{2\nu_0}\left[\frac{\cos^2\phi}{\sigma_x'^2} + \frac{\sin^2\phi}{\sigma_y'^2}\right].$$

Here ν_o is the value of ν in the forward direction and J_{max} is the determining parameter of the maximum opening angle of the photon beam.

For a given photon energy of $\hbar\omega$ the difference of the harmonic number,

$$\Delta\nu = \frac{K\Delta K}{1 + \frac{K^2}{2} + \gamma^2\theta^2}$$

depends on ΔK in the forward direction and becomes smaller as the opening angle θ is increased. It is assumed here the approximation of $\vec{n} = \vec{n}_o$, where $\vec{n}$ and $\vec{n}_o$ are unit vectors from th position of the electron beam to the observer and from the origin of the coordinate system to the observer, respectively (LS-8).

In the case of ideal undulators the radiated photon beam has a circular pattern with an angular interval

$$\gamma\Delta\theta = \frac{1 + K^2/2}{2(\gamma\theta)\nu_o}.$$

The angular interval becomes narrower as the harmonic order or the viewing angle becomes higher.

The width and the intensity variation of the circular pattern depend on the function

$$\left(\frac{\sin N\pi\nu}{\sin \pi\nu}\right)^2 .$$

The interference term, of course, is for the overlapping of the function of Eq. (5),

$$\left(\frac{\sin N_i\pi\nu_i}{\sin \pi\nu_i}\right)\left(\frac{\sin N_j\pi\nu_j}{\sin \pi\nu_j}\right).$$