Dependence of Undulator Spectra on Viewing Slits

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The undulator spectrum by a single electron (or positron) in a finite N-period undulator, with a periodic magnetic field of $B_y = B_o \sin(2\pi z/\lambda_u)$, may be expressed as

$$\frac{d^2I}{d\omega d\Omega} = \frac{4e^2\gamma^2N^2}{c}\left(\frac{\sin N\pi\nu}{N\sin\pi\nu}\right)^2 \quad g_\nu(p,q)\ ^2, \tag{1}$$

where ν is the harmonic number of the radiated photon energy and $g_\nu(p,q)$ is given in LS-8.[1] In Eq. (1), $g_\nu(p,q)$ is a rather slowly varying function and $(\sin N\pi\nu/N\sin\pi\nu)^2$ represents the "diffraction-like pattern" of the radiation. Since ν is a function of the angle θ in the spherical coordinate system of (r, θ, ψ) for a given photon energy of $h\omega$, the variation of an ideal single-electron undulator spectrum could be approximated as

$$f_i(\theta) = \left(\frac{\sin N\pi\nu}{N\sin\pi\nu}\right)^2. \tag{2}$$

For an n-electron beam with a Gaussian distribution of

$$f_e(\theta) = \frac{n}{2\pi\sigma_x'\sigma_y'}\, e^{-\frac{1}{2}\left(\frac{\theta_x^2}{\sigma_x'^2} + \frac{\theta_y^2}{\sigma_y'^2}\right)}, \tag{3}$$

the undulator spectrum should be proportional to the convolution of the functions $f_i(\theta)$ and $f_e(\theta)$,

$$f_{ie}(\theta) = \int_{-\infty}^{\infty} f_i(\theta')f_e(\theta-\theta')d\theta'. \tag{4}$$

The probability that the radiated photon beam passes through a slit may be expressed as

$$f_{ies}(\theta) = \int_{-\infty}^{\infty} f_{ie}(\theta)f_s(\theta)d\theta$$

$$= \int_{-\infty}^{\infty} f_i(\theta')\ f_{es}(\theta')d\theta', \tag{5}$$

where $f_s(\theta)$ is a probability function of the slit and $f_{es}(\theta)$ is the convolution of $f_e(\theta)$ and $f_s(\theta)$,

$$f_{es}(\theta) = \int_{-\infty}^{\infty} f_e(\theta-\theta')f_s(\theta')d\theta'. \tag{6}$$

For a perfect slit with a viewing angle of θ_{sx} and θ_{sy} in the x and y directions, respectively, the normalized function $f_s(\theta)$ is given by

$$f_s(\theta) = \frac{1}{4\theta_{sx}\theta_{sy}}. \tag{7}$$

In this case, the function $f_{es}(\theta)$ of Eq. (6) becomes

$$f_{es}(\theta) = \frac{1}{16\theta_{sx}\theta_{sy}}F(\theta_x)F(\theta_y), \tag{8}$$

where

$$F(\theta_n) = \mathrm{erf}\left(\frac{\theta_n + \theta_{sn}}{\sqrt{2}\sigma_n'}\right) - \mathrm{erf}\left(\frac{\theta_n - \theta_{sn}}{\sqrt{2}\sigma_n'}\right).$$

An alternative method is the double Fourier transformation of the "Dirichlet's discontinuous factor,"(2)

$$
\begin{aligned}
f(x) &= 1 \qquad \text{for} \qquad x < 1 \\
&= 1/2 \qquad\qquad\qquad\;\; = 1 \\
&= 0 \qquad\qquad\qquad\quad\;\; > 1.
\end{aligned}
$$

In this method, Eq. (7) becomes

$$f_s(\theta) = \frac{1}{\pi^2\theta_{sx}\theta_{sy}} \int_o^\infty \frac{\sin(\theta_{sx}\alpha)}{\alpha}\cos(\theta_x\alpha)d\alpha \int_o^\infty \frac{\sin(\theta_{sy}\beta)}{\beta}\cos(\theta_y\beta)d\beta. \qquad (9)$$

Then, the convolution function $f_{es}(\theta)$ is

$$f_{es}(\theta) = \frac{1}{4\pi^2\theta_{sx}\theta_{sy}} G(\theta_x)\, G(\theta_y), \qquad (10)$$

with

$$G(\theta_n) = \int_o^\infty e^{-\frac{1}{2}\sigma_n'^2\xi}\, \frac{\sin(\theta_{sn}+\theta_n)\xi + \sin(\theta_{sn}-\theta_n)\xi}{\xi} d\xi. \qquad (11)$$

Obtaining the undulator spectrum using Eqs. (8) and (10) is a rather tedious calculation even in the forward direction of $\theta = 0$. Two types of approximation of the slit, delta function and Gaussian distribution, are considered in the following.

For a slit with a pinhole ($\theta_s \ll \sigma_n'$) in the forward direction of $\theta = 0$, $f_s(\theta)$ may be approximated as a delta function of

$$f_s(\theta) = \delta(\theta_{sx})\delta(\theta_{sy}), \tag{12}$$

and Eq. (5) is reduced to

$$f_{ies}(0) = \int_{-\infty}^{\infty} f_i(\theta) f_e(\theta) d\theta. \tag{13}$$

If the probability function of the slit is approximated as a Gaussian distribution of

$$f_s(\theta) = \frac{1}{2\pi\theta_{sx}\theta_{sy}} e^{-\frac{1}{2}\left(\frac{\theta_x^2}{\theta_{sx}^2} + \frac{\theta_y^2}{\theta_{sy}^2}\right)}, \tag{14}$$

one obtains

$$f_{es}(\theta) = \frac{1}{2\pi\Sigma_x'\Sigma_y'} e^{-\frac{1}{2}\left(\frac{\theta_x^2}{\Sigma_x'^2} + \frac{\theta_y^2}{\Sigma_y'^2}\right)}, \tag{15}$$

where

$$\Sigma_n' = (\sigma_n'^2 + \theta_{sn}^2)^{1/2}. \tag{16}$$

Equation (16) shows that the one-standard deviations of the viewing slit, σ_{sx} and σ_{sy}, in the Gaussian approximation contribute additional broadening of the photon spectrum. It is equivalent to having an increased effective electron beam divergence in the pinhole approximation of the slit.

Two examples are shown in Figs. 1 and 2 for the dependence of the spectrum broadening due to the slit angles in the forward direction. In the case of the pinhole approximation, the following electron beam parameters are used: σ_x = 0.405 mm, σ_y = 0.098 mm, σ_x' = 0.018 mrad, and σ_y' = 0.007 mrad. As the θ_{sx} and θ_{sy} are increased by 0.01 mrad, the peaks of the harmonics are reduced by a half, and the corresponding photon energies are shifted toward lower values.

References

1. S. H. Kim, "Calculation of the Undulator Radiation Spectra," Light Source Note, LS-8 (January 1985)

2. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. 1, Interscience Publishers, Inc., New York (1953). p 81.

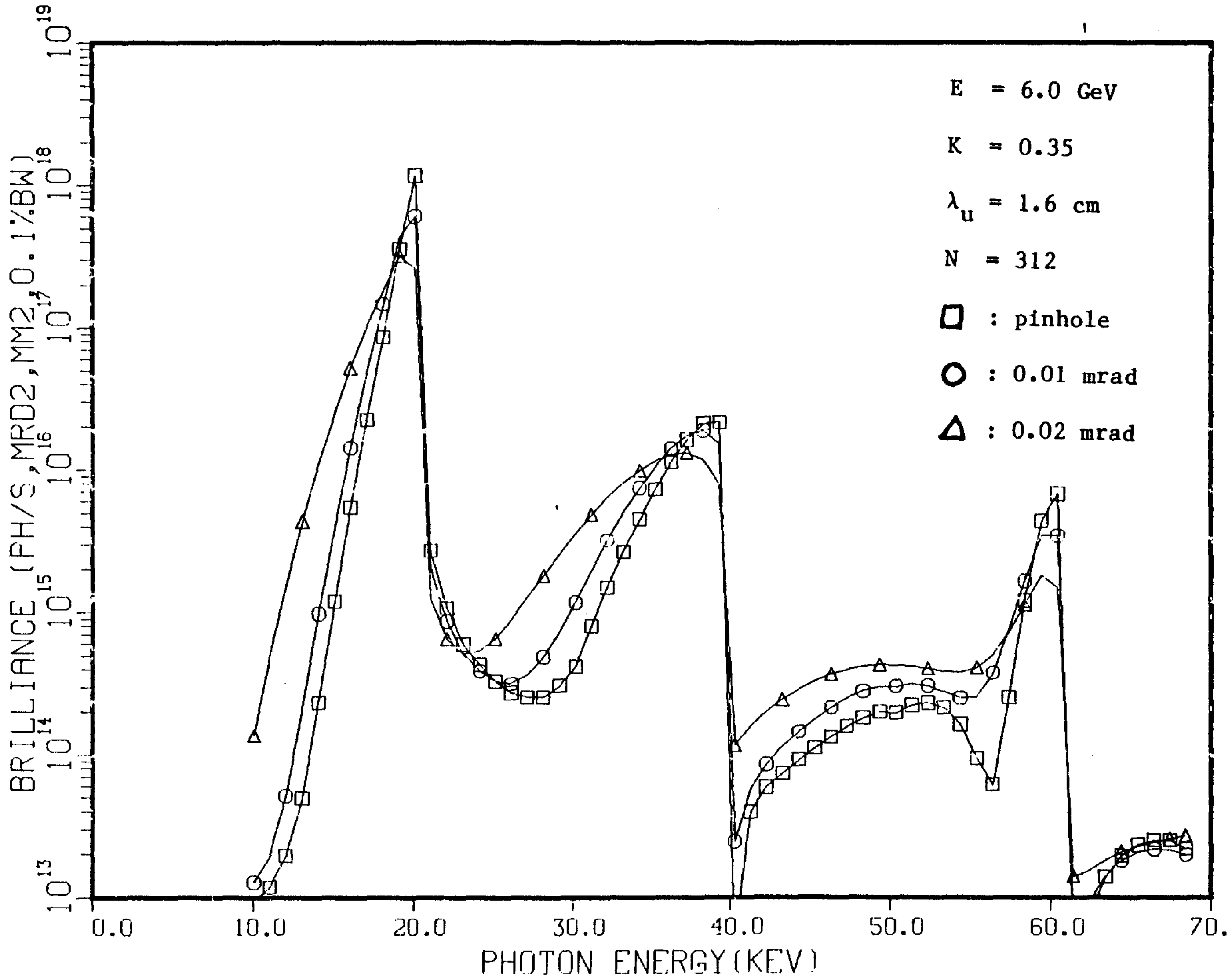

Figure 1, 20-keV undulator spectra. Data points with squares are for the case of pinhole slit, and those with circles and triangles are for 0.01 mrad and 0.02 mrad of θ_{sn}, respectively, in Gaussian approximation.

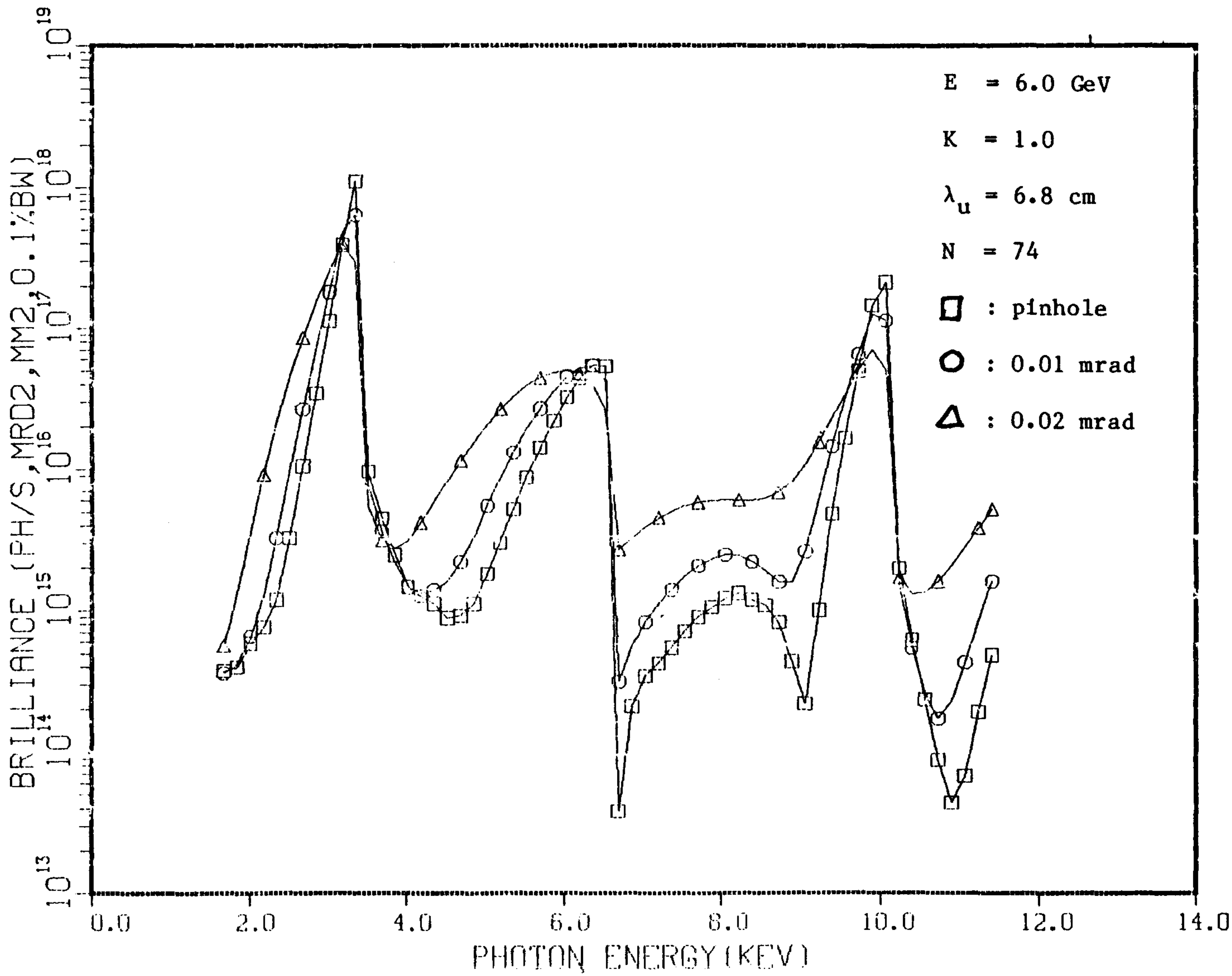

Figure 2. 3 ~ 10-keV undulator spectra. Data points with squares are for the case of pinhole slit, and those with circles and triangles are for 0.01 mrad and 0.02 of θ_{sn}, respectively, in Gaussian approximation.