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A Radial Coil Probe for Quadrupole Magnet Measurements

Introduction

This note describes a rotating coil probe of "radial-winding geometry" for the measurements of the magnetic center, quadrupole fields and multipole coefficients of quadrupole magnets. The active length of the coil is longer than the magnet length, so that all the measurements will be integrated values along the longitudinal direction of the magnets. Errors of the measurements due to the fabrication tolerances of the coil are discussed.

Multipole Coefficients

The magnetic field in the aperture of a magnet may be expressed in the two-dimensional Cartesian or cylindrical coordinate system:

$$B_y + iB_x = B_o \sum_{m=o}^{\infty} (b_m + ia_m)(x + iy)^m, \tag{1}$$

$$B_\theta + iB_r = (B_y + iB_x)\exp(i\theta)$$

$$= \sum_{n=1}^{\infty} C_n \exp(-in\alpha_n)\left[\frac{Z}{R}\right]^{n-1} \exp(i\theta), \tag{2}$$

where $z = x + iy = r \exp(i\theta)$,

and the multipole coefficients for the two different definitions are related by

$$\frac{C_n}{R^{n-1}} \exp(-in\alpha_n) = B_o(b_{n-1} + ia_{n-1}). \tag{3}$$

Here R is a reference radius and $b_1 = 1.0\ cm^{-1}$ for quadrupole magnets. The reference angle for a normal quadrupole magnet is defined as $a_1 = 0$ or $\alpha_1 = 0$. For the storage ring quadrupole Q4 of the APS, $B_o b_1 = C_2/R = 18.9$ T/m at 7 GeV.

For a quadrupole magnet with effective length L, the total flux linkage for coil A at an angular position θ in Fig. 1 is

$$\Phi_A(\theta) = \sum_{n=1}^{\infty} L N_A C_n \frac{1}{nR^{n-1}} \left[r_{A2}^{\ n} - (-r_{A1})^n \right] \cos n(\theta - \alpha_n). \tag{4}$$

When the two probe coils, coil A and coil B in Fig. 1, are connected in series such that their flux linkages are opposed to each other, the measurement sensitivity of higher multipoles, n>2, could be increased by rejecting the dipole and main (quadrupole) field components of the flux linkages.

The flux linkage for the two coils at θ is

$$\phi_{AB}(\theta) = \sum_{n=1}^{\infty} N_A C_n \frac{1}{nR^{n-1}} r_{A2}^{\ n} R_{fac} \cos n(\theta - \alpha_n), \tag{5}$$

where

$$R_{fac} = 1 - (-r_{A1}/r_{A2})^n - (N_B/N_A)\ [r_{B2}^{\ n} - (-r_{B1})^n]/r_{A2}^{\ n} \tag{6}$$

represents the sensitivity of the measurements for each multipole coefficient. From Eq. (6), the conditions for the rejection of dipole and quadrupole components are given by

$$r_{A2} + r_{A1} = (N_B/N_A)\ (r_{B2} + r_{B1}),$$

$$r_{A2} - r_{A1} = r_{B2} - r_{B1}. \tag{7}$$

Table 1 lists two examples of the coil locations for $N_B/N_A = 2$ and their R_{fac}. Each multipole coefficient obtained from the Fourier transformation of the measured data should be divided by the measurement sensitivity, R_{fac}, listed in Table 1 in order to obtain the relative magnitude of the multipole coefficient at radius r_{A1}.

Table 1. Coil Locations and R_{fac}

Parameter	Case #1	Case #2
N_B/N_A	2	2
r_{A2}	r_{A2}	r_{A2}
r_{A1}	$0.5\ r_{A2}$	$0.55\ r_{A2}$
r_{B2}	$0.625\ r_{A2}$	$0.6125\ r_{A2}$
r_{B1}	$0.125\ r_{A2}$	$0.1625\ r_{A2}$
n	R_{fac}	R_{fac}
1	0.000000	0.000000
2 (quadrupole)	0.000000	0.000000
3	0.632812	0.698226
4	0.632812	0.628403
5	0.840454	0.877692
6	0.865173	0.866755
7	0.933305	0.950537
8	0.949527	0.952010
9	0.972849	0.980340
10	0.980833	0.982604
11	0.989119	0.992289
12	0.992650	0.993658
13	0.995681	0.997006
14	0.997163	0.997676
15	0.998295	0.998846

Magnetic Center and Quadrupole Field Integral

Figure 2 shows two coordinate systems, the xy-coordinate system with its origin at the magnetic center (MC) and the x'y' at the cylinder rotation axis (CR). The location of the CR with respect to the MC is $Z_o = r_o\exp(i\theta_o)$. If the x'y'-coordinate system is displaced from the xy-coordinate system, the expression of the magnetic field at point, p, with respect to the two coordinate systems can be expressed as

$$\sum_{J=1}^{\infty} C'_J \exp(-iJ\alpha'_J)\left(\frac{Z'}{R}\right)^{J-1} = \sum_{n=1}^{\infty} C_n \exp(-in\alpha_n)\left(\frac{Z}{R}\right)^{n-1}, \qquad (8)$$

where $Z' = r'\exp(i\theta')$

$Z = r\exp(i\theta)$

$= Z' + Z_o = Z'(1 + Z_o/Z')$.

By using the following relation,

$$(1 + a)^{n-1} = \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!} a^{k-1},$$

Eq. (8) becomes

$$\sum_{J=1}^{\infty} C_J' \exp(iJ\alpha'_J)\left(\frac{Z'}{R}\right)^{J-1} = \sum_{J=1}^{\infty}\sum_{N=J}^{\infty} C_N \exp(-iN\alpha_N)\frac{(N-1)!}{(J-1)!(N-J)!}\left(Z_o/R\right)^{N-J}\left(\frac{Z'}{R}\right)^{J-}. \qquad (9)$$

Equation (9) shows that the dipole field component in the x'y'-coordinate system consists of the dipole and all other higher multipole components in the xy-coordinate system. If we write up to the sextupole terms of both sides of Eq. (9), we find

$$\sum_{J=1}^{3} C'_J \exp(-iJ\alpha'_J)\left(Z'/R\right)^{J-1} = C_1\exp(-i\alpha_1) + C_2\exp(-i2\alpha_2)\left(Z_o + Z'\right)/R + C_3\exp(-i3\alpha_3)\left(Z_o + Z'\right)^2/R^2 . \quad (10)$$

For a typical quadrupole magnet, $C_n\ (n\neq2)/C_2 < 1$, the azimuthal field component in the x'y'- coordinate system due to the main quadrupole field only is

$$B'_\theta = \mathrm{Re}\{C_2\exp(-i2\alpha_2 + i\theta')\left(Z_o + Z'\right)/R\} . \quad (11)$$

The flux linkage for coil A in Fig. 1 is

$$\phi'_A(\theta) = LN_A \frac{C_2}{R}\left[r_o\ (r_{A2}+r_{A1})\ \cos(\theta'+\theta_o-2\alpha_2) + \frac{r_{A2}{}^2-r_{A1}{}^2}{2}\cos2(\theta'-\alpha_2)\right] . \quad (12)$$

The off-center of the cylinder rotation axis with respect to the magnetic center can be found from the first term of Eq. (12). The second term of Eq. (12) gives the quadrupole field integral, which does not depend on the off-axis distance Z_o. When Z_o is not zero, it should be noted, however, that there are other correction terms in the quadrupole field integral measurements such as the $2C_3\exp(-i3\alpha_3)\ Z_oZ'/R^2$ term of Eq. (10).

For L = 0.6m, N_A = 20 turns, C_2/R = 19 T/m, $r_{A2} + r_{A1}$ = 50 mm and ω = 3 for $\theta = \omega t$, the induced voltage from coil A is

$$V_A(t) = 35(\mathrm{mV})\, r_o(\mu\mathrm{m})\sin(\omega t+\theta_o-2\alpha_z) + 0.6(\mathrm{V})\sin2(\omega t-\alpha_z) . \quad (13)$$

Equations (12) and (13) indicate that the magnetic center-detecting coil should have radii of $r_{A2} = r_{A1}$ to minimize the $\sin(2\omega t)$ term of the equations.

Coil Position Errors

In Fig. 3, it is assumed that the plane of coil A is displaced from the cylinder rotation axis by Δr. When the angular position of r_{A2} is used as the reference angular position for the measurements, the errors of radial and angular positions of r_{A2} and r_{A1} are $(\Delta r_A, 0)$ and $(\Delta r_A, \Delta\theta)$, where $\Delta\theta = \Delta r\,(r_{A2} + r_{A1})/r_{A2}\,r_{A1}$. Then the flux linkage of coil A is given by

$$\phi_A(\theta) = LN_A(D + Q), \tag{14}$$

where

$$D = \frac{C_2}{R}\, r_o(r_{A2} + r_{A1}) \left[\left(1 + \frac{\Delta r + 2\Delta r_A}{r_{A2} + r_{A1}}\right) \cos(\theta' + \theta_o - 2\alpha_2) + \frac{\Delta r}{r_{A2}} \sin(\theta' + \theta_o - 2\alpha_2)\right],$$

$$Q = \frac{C_2}{2R} (r_{A2}{}^2 - r_{A1}{}^2) \left[1 + \frac{\Delta r + 2\Delta r_A}{r_{A2} + r_{A1}} + \Delta r \frac{r_{A1}}{r_{A2}} \frac{1}{r_{A2} + r_{A1}} \sin 2(\theta' - \alpha_2)\right].$$

In Eq. (14) the D and Q terms are used for the measurements of the magnetic center axis and quadrupole field integral, respectively. Relative errors, $\Delta r/r_{A2}$, $\Delta r_2/r_{A2}$ and $\Delta r_1/r_{A2}$, in Eq. (14), do not make the procedures for the detection of the magnetic center axis particularly difficult. In the Fourier analysis of the measurement data, however, one should keep in mind the angular phase error due to $\sin\theta'$ term. For a relative error of radial positions of 1×10^{-3} (for case #1 coil geometry in Table 1, $\Delta\theta = 3 \times 10^{-3}$), the error of the Q term in Eq. (14) would be less than 5×10^{-3}.

Relative errors of multipole coefficients due to coil position tolerances for coil A and coil B in Table 1 are listed in Table 2. It is assumed that relative radial position errors of the coils with respect to r_{A2} and relative position errors of the coil planes from its rotation axis are 1×10^{-3}.

Table 2. Multipole Coefficient Errors

n	Relative errors
1	6×10^{-3}
2	7×10^{-3}
3	7×10^{-3}
4	7×10^{-3}
6	9×10^{-3}
10	15×10^{-3}

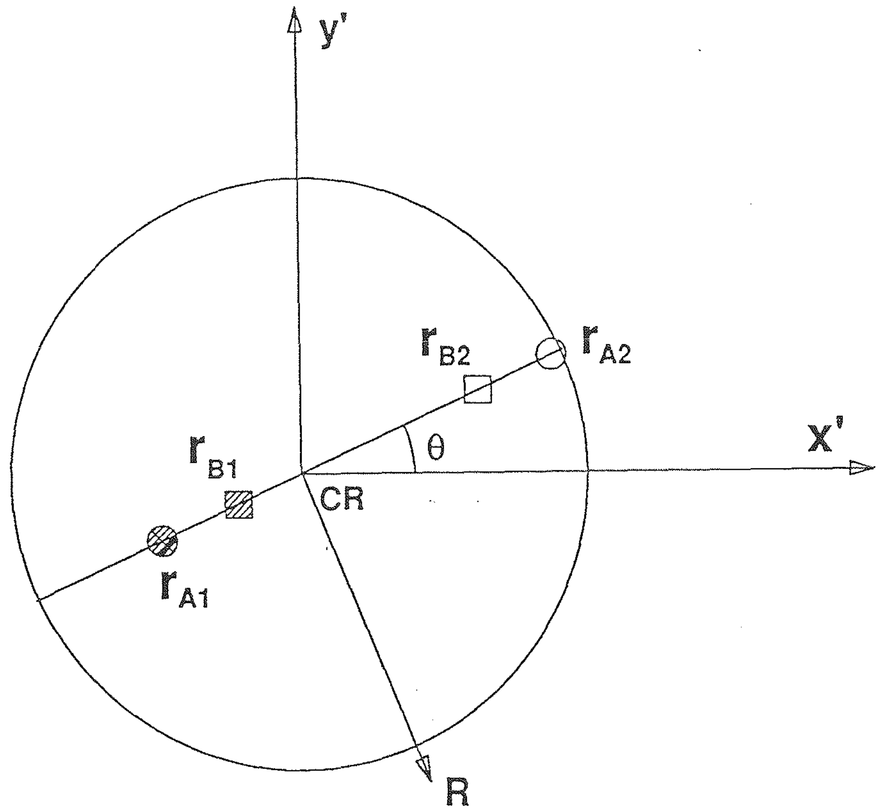

Fig. 1. Cross section of two sets of radial winding coils in a cylinder. Coil A is located at (r_{A2}, θ) and (r_{A1},$\theta+\pi$), and has a number of turns N_A. Coil B is located at (r_{B2},θ) and (r_{B1},$\theta+\pi$) with N_B turns.The cylinder rotation axis (CR) is located at the origin of the x'y'-coordinate system. The CR is in the plane of the two coils. The radius of the measuring magnet aperture can be chosen as the reference radius R. The reference angular position, $\theta = 0$, is the direction of the CR/r_{A2}.

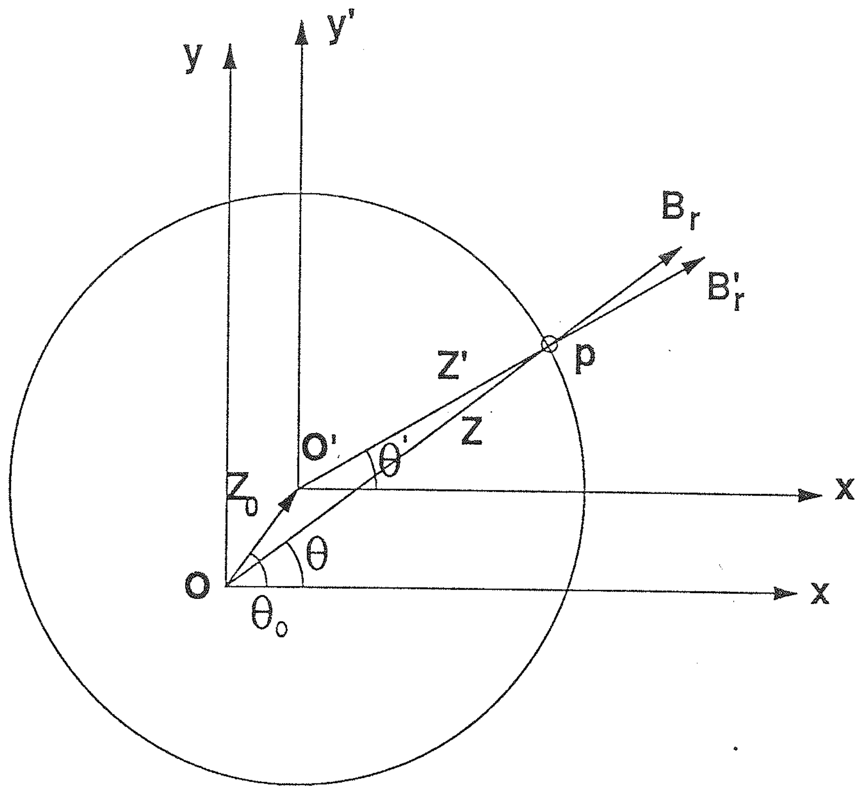

Fig.2. Coordinate systems of the magnetic center (MC) and cylinder rotation axis (CR). The MC is located at O. The CR,which is located at O',is displaced from MC by $Z_o = r_o \exp(i\theta_o)$.The corresponding axes of the xy and x'y' coordinates are in parallel. Z and Z' are coordinates of point P on the cylinder surface with respect to the two coordinate systems.

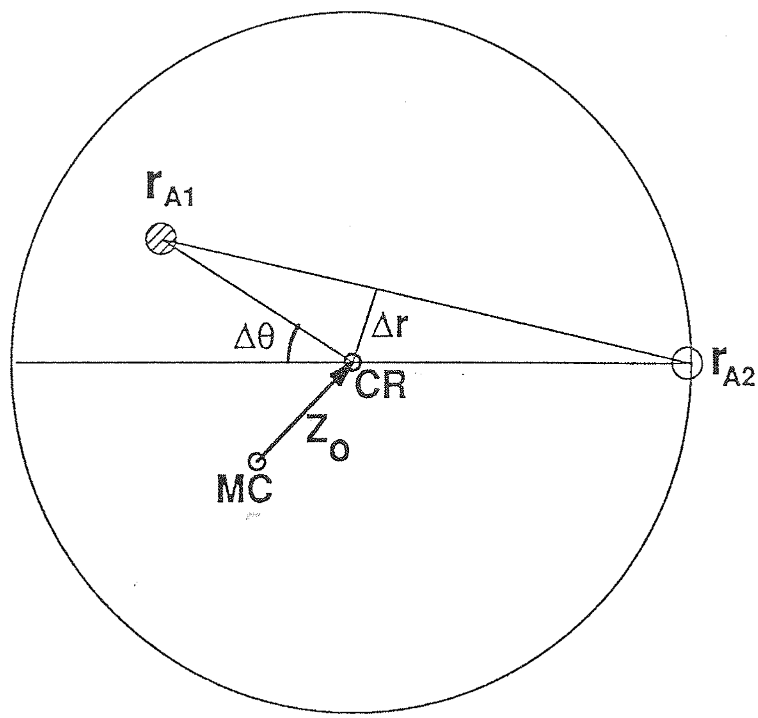

Fig.3. Position error of coil A with respect to the CR. The plane of coil A is displaced from the CR by Δr. Using the angular position of r_{A2} is used as the reference angular position, $\Delta\theta$ is shown as the angular position error of r_{A1}.