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The Effect of the Residual Gas on the Beam Life Time in Electron (Positron) Storage Ring

The beam loss due to the residual gas is mainly determined by bremsstrahlung and elastic single scattering with the atoms of the residual gas. It is convenient to introduce a cross section for each beam loss process: $\sigma(B)$ for bremsstrahlung and $\sigma(S)$ for single scattering. The effective beam lifetime is given by

$$\frac{1}{\tau_{eff}} = c \sum_i N_i \left(\sigma_i(B) + \sigma_i(S)\right)$$

$$= \sum_i \left(\frac{1}{\tau_i(B)} + \frac{1}{\tau_i(S)}\right) ,$$

where c = velocity of light

N_i = number of atoms i per unit volume.

In general, the residual gas contains mainly a mixture of H_2 and CO so that i stands for H_2 or CO.

For a vacuum of 10^{-8} mm Hg or lower, the distance between molecules is much larger than the radius of the atom. This is also true for the beam particles. Therefore, the probability of head-on collisions is very small and can be neglected. At high energies, the cross section for the production of a photon with energy between $h\nu$ and $h(\nu + d\nu)$ by an incident electron (positron) of energy E is given by (ref. 1):

$$\phi(E,\nu)d\nu = \frac{4\, Z^2 r_e^{\,2}}{137} \left[\left\{1 + \left(\frac{E - h\nu}{E}\right)^2 - \frac{2}{3}\,\frac{E - h\nu}{E}\right\} \ell n \frac{183}{Z^{1/3}} + \frac{1}{9}\,\frac{E - h\nu}{E}\right] \frac{d\nu}{\nu}$$

$$\approx \frac{4Z^2 r_e^{\ 2}}{137}\ \{1 + (\frac{E - h\nu}{E})^2 - \frac{2}{3}\ \frac{E - h\nu}{E}\}\ \ell n\ \frac{183}{Z^{1/3}}\ \frac{d\nu}{\nu} \tag{1}$$

This equation contains only the radiation in the field of the atomic nucleus. To include the effect of the atomic electrons, Z^2 must be replaced by $Z(Z + \zeta)$, where ζ is given by (see ref. 2)

$$\zeta = \frac{\ell n\ 1440/Z^{2/3}}{\ell n\ 183/Z^{1/3}}\ .$$

When the radiated energy $h\nu$ is larger than the bucket height ΔE_{rf}, the particle will be lost. Integration of Eq. (1) from $h\nu = \Delta E_{rf}$ to $h\nu = E$ and replacing Z^2 by $Z(Z + \zeta)$, one obtains

$$\sigma(B) = \frac{4\ r_e^2}{137}\ Z(Z + \zeta)\ (\frac{4}{3}\ \ell n\ \frac{E}{\Delta E_{rf}} - 5/6])\ell n\ \frac{183}{Z^{1/3}} \tag{2}$$

The differential cross section for scattering by the nucleus and the atomic electrons through the angle θ into the solid $2\pi\ \sin\theta\ d\theta$ can be written in the form

$$d\sigma = \frac{2\pi\ Z(Z+1)}{4\gamma^2}\ \frac{\sin\theta\ d\theta}{\sin^4\theta/2}$$

$$\approx \frac{8\pi\ Z(Z + 1)\ d\theta}{\gamma^2\ \ \ \ \theta^3}$$

The particle will probably be lost if $\bar{\theta}^2$ is larger than $\frac{h_m^{\ 2}}{\beta_m \bar{\beta}}$, where $2h_m$ is the minimum aperture of the vacuum chamber, β_m is the value of the beta function at the location of h_m and $\bar{\beta}$ is the average value of the beta function. Performing the integration gives

$$\sigma(S) = \frac{4\pi\ Z(Z + 1)}{\gamma^2}\ \frac{\beta_m \bar{\beta}}{h_m^{\ 2}} \tag{3}$$

Substituting Eqs. (2) and (3) in

$$\frac{1}{\tau_i(B)} = cN_i \ \sigma_i(B) \text{ and } \frac{1}{\tau_i(S)} = cN_i\sigma_i(S)$$

one finds

$$\frac{\tau(S)}{\tau(B)} = \frac{1.45 \times 10^{-2} \left(\frac{4}{3} \ln \frac{E}{\Delta E_{rf}} - 5/6\right) h_m^2 \gamma^2}{\beta_m \bar{\beta}} \text{ for } H_2 \quad (Z = 1)$$

and

$$\frac{\tau(S)}{\tau(B)} = \frac{1.1 \times 10^{-2} \left(\frac{4}{3} \ln \frac{E}{\Delta E_{rf}} - 5/6\right) h_m^2 \gamma^2}{\beta_m \ \bar{\beta}} \text{ for CO} \quad (Z = 7)$$

Examples

6-GeV light sources

Assumptions: minimum apertures at undulators $h_m = h_u = 5$ mm, $\beta_u = 20$ m,

$\bar{\beta} = 10$ m, $E/\Delta E_{rf} = 125$ or $\frac{\Delta E_{rf}}{E} = 0.8\%$

H_2 : $\frac{\tau(S)}{\tau(B)} = 1.4$; CO : $\frac{\tau(S)}{\tau(B)} = 1.06$

Aladdin Electron energy 800 MeV

Assumptions: $h_m = 5$ mm, $\beta_m = 4$ m, $\bar{\beta} = 2$ m and $E/\Delta E_{rf} = 125$ or $\frac{\Delta E_{rf}}{E} = 0.8\%$

$H_2 = \frac{\tau(S)}{\tau(B)} = 0.62$; CO : $\frac{\tau(S)}{\tau(B)} = 0.47$

References

1. H. A. Bethe and J. Ashkin, Experimental Nuclear Physics, Vol. 1, p. 260.
2. H. A. Bethe and J. Ashkin, Experimental Nuclear Physics, Vol. 1, p. 263.