6 GeV Light Source Storage Ring
Quadrupole and Sextupole Magnet Field Calculations

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Quadrupole Magnet

Figure 1 shows the cross section of one-fourth of a storage ring quadrupole magnet. The vacuum chamber outline is shown by the dashed line. A tapered pole is necessary to reduce the pole saturation. This makes the coil winding more complicated but is essential to reduce the power of the magnet. Details of the x, y coordinates of the pole tip and shim are also given. This geometry was transformed into a dipole magnet in the u-v plane using the equations:

$$u = \frac{x^2 - y^2}{2R_o}; \qquad v = \frac{xy}{R_o}$$

Figure 2 shows a flux plot of this geometry in the u-v plane. Notice that the 90 degree section of the quadrupole transforms to a 180 degree section of the dipole. An enlarged view of the tapered pole is also shown. Symmetry exists only in the midplane so all harmonics are possible for the dipole and therefore all even harmonics are possible for the quadrupole. The field for the quadrupole can be written as:

$$B = B_2\left(\frac{r}{r_o}\right) + B_4\left(\frac{r}{r_o}\right)^3 + B_6\left(\frac{r}{r_o}\right)^5 + B_8\left(\frac{r}{r_o}\right)^7 = B_{10}\left(\frac{r}{r_o}\right)^9 + \cdots$$

It turns out that the B_4, B_8, B_{12}, $\cdots$ are all less than 1 gauss at a radius of 3 cm so the terms that remain are:

$$B = B_2\left(\frac{r}{r_o}\right) + B_6\left(\frac{r}{r_o}\right)^5 + B_{10}\left(\frac{r}{r_o}\right)^9 + \cdots \qquad (1)$$

which are the terms for a quadrupole that has symmetry about the 45 degree line. The coil and pole of the quadrupole are symmetric about the 45 degree line, only the yoke is not. Since the yoke is far removed from the bore, it does not affect the symmetry significantly. The B_n coefficients are calculated by POISSON and shown in Table I. The ampere turns per pole for the current multiplier of 1.0 was 12732.4. B_{10} has the largest value of -2.3 gauss out of 5715 gauss or -0.04 percent at a radius of 3 cm. From equation 1, the field due to this harmonic can be calculated at any other radius and is:

$$B_{10}\left(\frac{2}{3}\right)^9 = .026\ B_{10} = .06 \text{ gauss}$$

at a radius of 2 cm.

The nominal gradient of the magnet is the field at the pole 7619.7 divided by the bore radius of 4 cm or 1905 gauss/cm. The efficiency is obtained by dividing the field at the pole by what would be required to drive the gap. The maximum field in the pole of 18.4 kG and the leakage flux reduces the efficiency to 0.952. The yoke field is sufficiently small and does not affect the efficiency significantly. The stored energy of 732 joules per meter of magnet length is only for the one-fourth of the magnet calculated. Multiply by 4 for the total magnet times the effective length.

The field on the midplane of the magnet can also be expanded into a Maclaurin series.

$$B = a_1 x + \frac{a_3}{3!} x^3 + \frac{a_5}{5!} x^5 + \cdots \qquad (2)$$

These coefficients can be calculated by equating terms of each series. Hence setting x equal to r yields:

$$a_1 = B_2/r_o \ (G/cm)$$

$$a_3 = 3!B_4/r_o^3 \ (G/cm^3)$$

$$a_5 = 5!B_6/r_o^5 \ (G/cm^5)$$

$$\vdots$$

$$a_n = n!B_{n+1}/r_o^n \ (G/cm^n)$$

The last five rows in Table I give these coefficients.

As the current multiplier is increased from 1.0 to 1.4 to 1.9 to 3.0, the B_n becomes larger because the pole tip shim and the pole itself starts saturating. Even at 3.0, the sixth harmonic field of -41.9 gauss is only 0.52 percent of the field at a radius of 3 cm. A plot of this saturation effect is shown in Fig. 3.

Three different geometrics are plotted in Fig. 3. The upper curve is the geometry calculated above. It has a pole width of 5.6 cm at the pole tip and widens to 10.0 cm at the pole base as shown in Fig. 1. The pole length is 18.1 cm and the yoke is 10.0 cm wide, perpendicular to the flux. These dimensions are given in Table II. It is the most efficient geometry of the three cases.

The middle curve has a constant pole width of 5.6 cm instead of the taper. This design requires 1.8 times the nominal current to obtain 2000 gauss/cm. Hence the efficiency is 0.56. The lower curve has a constant pole width of 6.6 cm. Since much more flux enters the pole tip relative to

the 1 cm extra width, it saturates more rapidly and requires three times the nominal current to obtain 2000 gauss/cm. The efficiency is only 0.33. This readily demonstrates the need for the tapered pole.

The average field in the 10 cm thick yoke at the top is about 13 kG for a current multiplier of 1.0. At the root of the pole (i.e., at the upper end of the coil), it is also 13 kG. The maximum field in the pole is 18.5 kG except in the shim where the average field is 20.2 kG. A practical coil has been designed by Ken Thompson, who also costed this magnet with Walter Praeg. See their memos for details.

Sextupole Magnet

Figure 4 shows the cross section of one-fourth of a storage ring sextupole magnet. The vacuum chamber outline is shown by the dashed line. Some modification of the chamber will be required. It was desired to have a 4 cm bore to keep the power at a reasonable level. A long narrow pole was needed to allow sufficient room for a low power coil. The details of the coil area, length of the pole, and shape of the yoke will be worked out by Ken Thompson when he designs a practical coil. The length of the pole from the tip to the root at the yoke may be much smaller and the side yokes may be vertical to allow supporting members for the top and bottom poles.

The magnet shown in Fig. 4 could have harmonics of 3, 6, 9, 12, 15, ••• However, because the magnet is almost symmetrical about the 30 degree line, with only the outer yoke not symmetrical, the 6, 12, ••• harmonics should be small, as the quadrupole calculations showed. Hence a 30 degree section of the magnet was calculated by POISSON. This was first transformed into the u-v plane dipole magnet using:

$$u = \frac{x^3 - 3xy^2}{3R_o^2}; \qquad v = \frac{-y^3 + 3x^2y}{3R_o^2}$$

This is shown as a flux plot in Fig. 5. The pole tip shape can be described by the equation:

$$3x^2y - y^3 = R_B^3$$

where R_B is the radius to the pole tip. With this shape pole the vacuum chamber limits the pole to 1.76 cm wide. Transformed into the u-v plane, this gives a ratio of pole width to gap height of only 0.72 which is a very difficult dipole magnet to shim to get a uniform field. Without shims, the field harmonics are given in Table III. The 9th harmonic of -79.4 gauss is 6.7 percent of the desired 3rd harmonic 1182. gauss at a radius of 3 cm. This pole shape already interferes slightly with the vacuum chamber.

If the pole tip is extended further towards the origin, it becomes narrower because of the vacuum chamber limitation and makes shimming more difficult. The only choice for shimming this magnet is to remove the central part of the pole tip along the 30 degree line; a negative shim. This results in the geometry shown in Fig. 4 and what is called the "All Shim Magnet," of Table III. The 9th harmonic of 1.7 gauss is 0.14 percent of the desired 3rd harmonic 1217. gauss at a radius of 3 cm for 1010 steel. The 15th harmonic increases to -35.5 gauss or 2.9 percent of the 3rd but should cause no problems since it decreases to 0.12 gauss at a radius of 2 cm.

The efficiency of the "All Shim Magnet," is 0.966, and drops to 0.730 for a current multiplier of 3. Since 1/12 of the magnet was solved, the stored energy must be multiplied by 12 and the effective length to get the total energy. After a practical coil is designed, the inductance of the magnet can be calculated from the stored energy.

The last four lines of Table III give the coefficients of a Maclaurin series.

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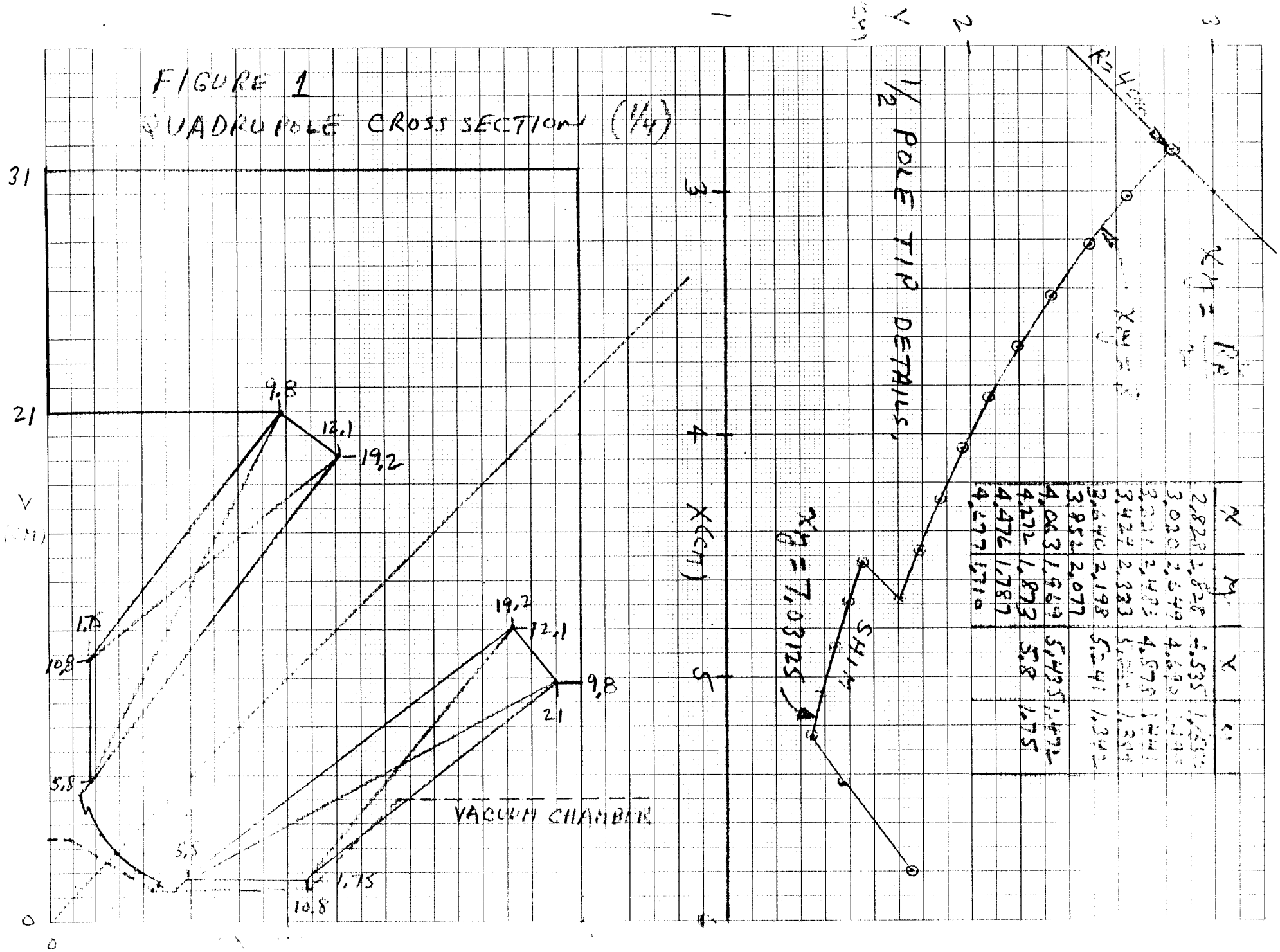
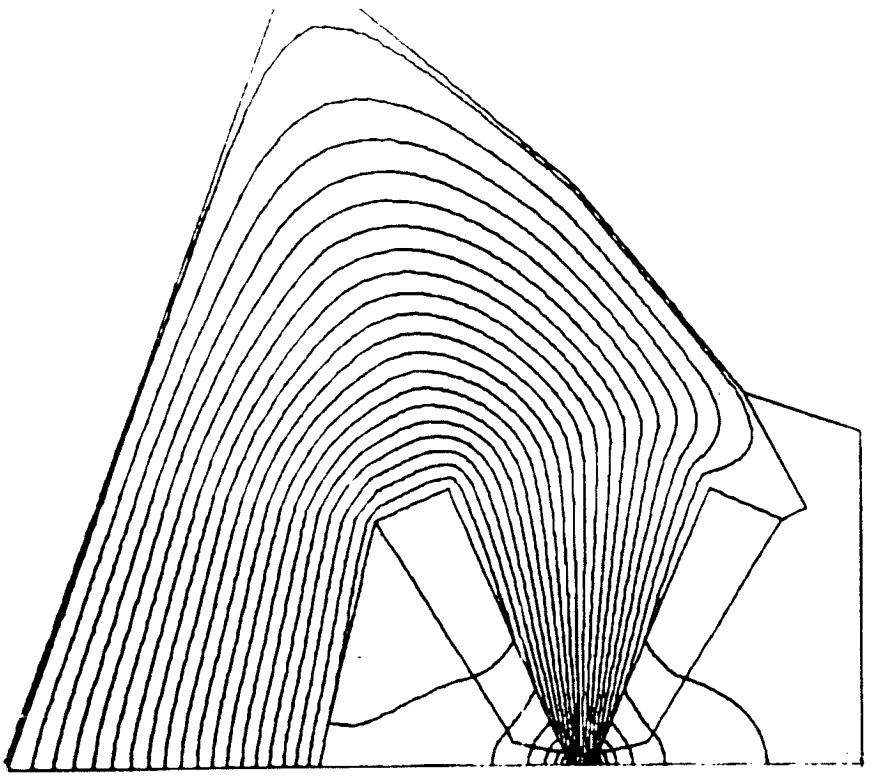


x	y	x	y
2.828	2.828	4.535	1.5[illegible]
3.020	2.649	4.690	[illegible]
3.221	2.482	4.578	[illegible]
3.427	2.333	[illegible]	1.354
3.640	2.198	5.241	1.342
3.852	2.077		
4.063	1.969	5.435	1.472
4.272	1.873	5.8	1.75
4.476	1.787		
4.677	1.710		

Figure 2. u-v Plane Flux Plot.



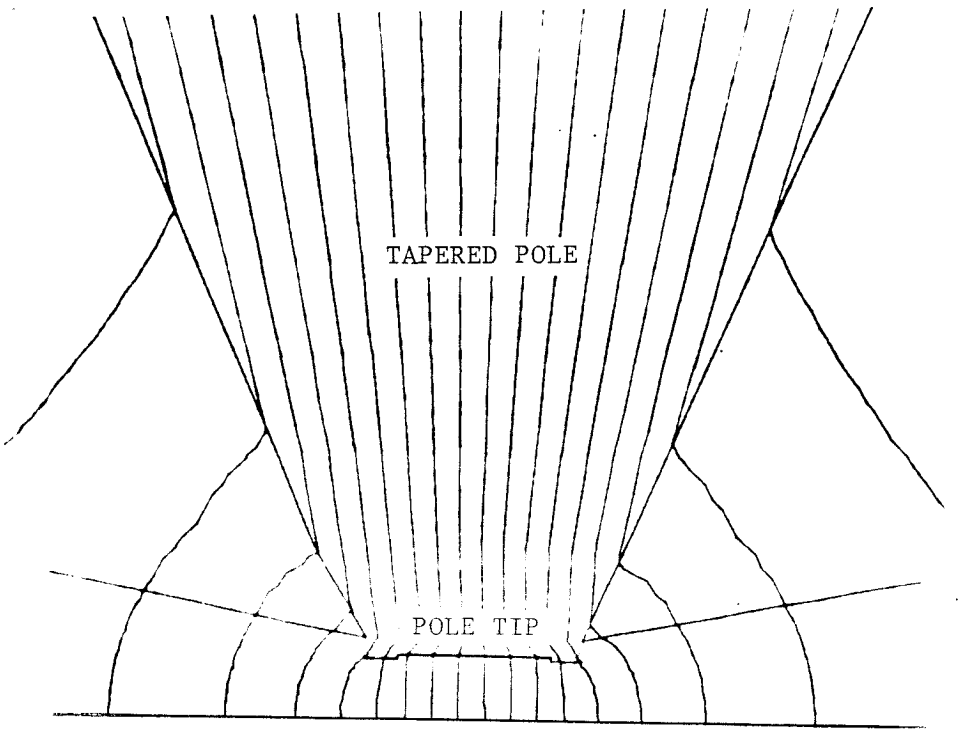


Table I

Storage Ring Quadrupole

Current Multiplier		($\mu = \infty$) 1.0	1010 Steel Fully Annealed 1.0	1.4	1.9	3.0
Field at the Pole		8004.0	7619.7	9111.3	9886.7	10808.0
B_n Harmonic Fields at R = 3 cm (Gauss)	6	3.9	-0.6	-13.7	-24.7	-41.9
	10	-1.6	-2.3	-5.0	-7.1	-9.9
	14	-1.0	-0.9	-1.0	-1.0	-1.0
	18	-0.2	-0.2	-0.1	-0.1	-0.1
Efficiency		1.000	.952	.813	.650	.450
Max Pole Field (kG)		--	18.4	21.7	23.2	24.7
Yoke Field (kG)		--	13.1	15.5	17.0	18.5
Stored Energy (Joules/M)		770.	732.	1225.	1811.	3137.
a_1(G/cm)		2001.	1905.	2278.	2472.	2702.
a_5(G/cm^5)		1.9	-0.3	-6.8	-12.2	-20.7
a_9(G/cm^9)		-29.5	-42.4	-92.2	-130.9	-182.5
a_{13}(G/cm^{13})		-3.9×10^3	-3.5×10^3	-3.9×10^3	-3.9×10^3	-3.9×10^3
a_{17}(G/cm^{17})		-5.5×10^5	-5.5×10^5	-2.8×10^5	-2.8×10^5	-2.8×10^5

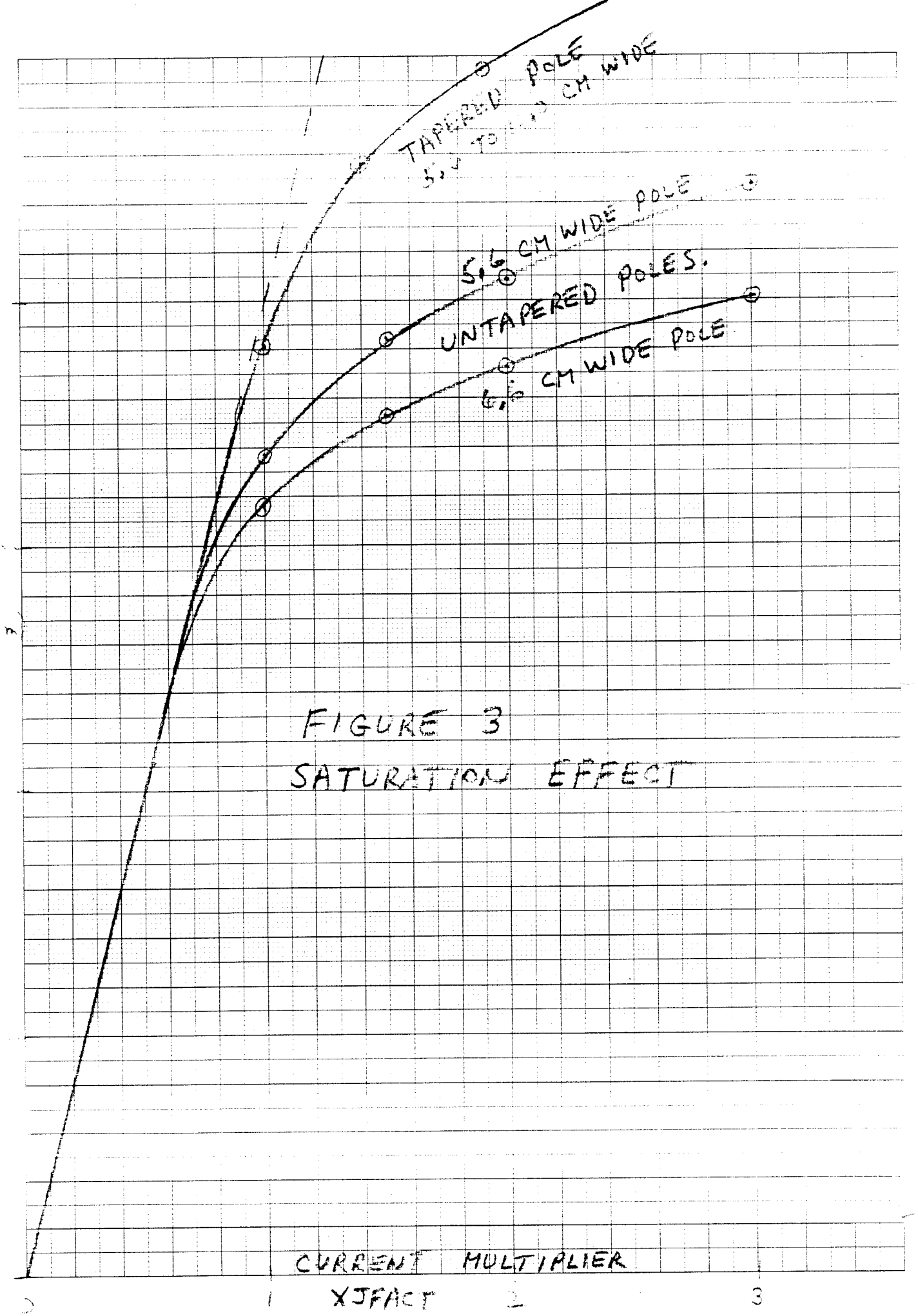


FIGURE 3
SATURATION EFFECT

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Table II

Storage Ring Quadrupole

Current Multiplier	Case, as shown in Figure 3		
	Bottom	Middle	Top
1.0 ($\mu = \infty$)	2000	2001	2001
1.0	1578	1683	1905
1.4	--	--	2278
1.5 Field Gradient	1765	1919	--
1.9 (G/cm) 1010 Steel	--	--	2472
2.0	1862	2043	--
3.0	2009	2238	2702
Pole width at tip (cm)	6.60	5.60	5.6
Pole width at base (cm)	6.60	5.60	10.0
Pole length (cm)	18.0	15.6	18.1
Yoke width (cm)	16.9	8.5	10
Area of Coil (cm^2)	45.9	46.4	46.2
j_o (amps/cm^2)	277.4	275.6	275.6

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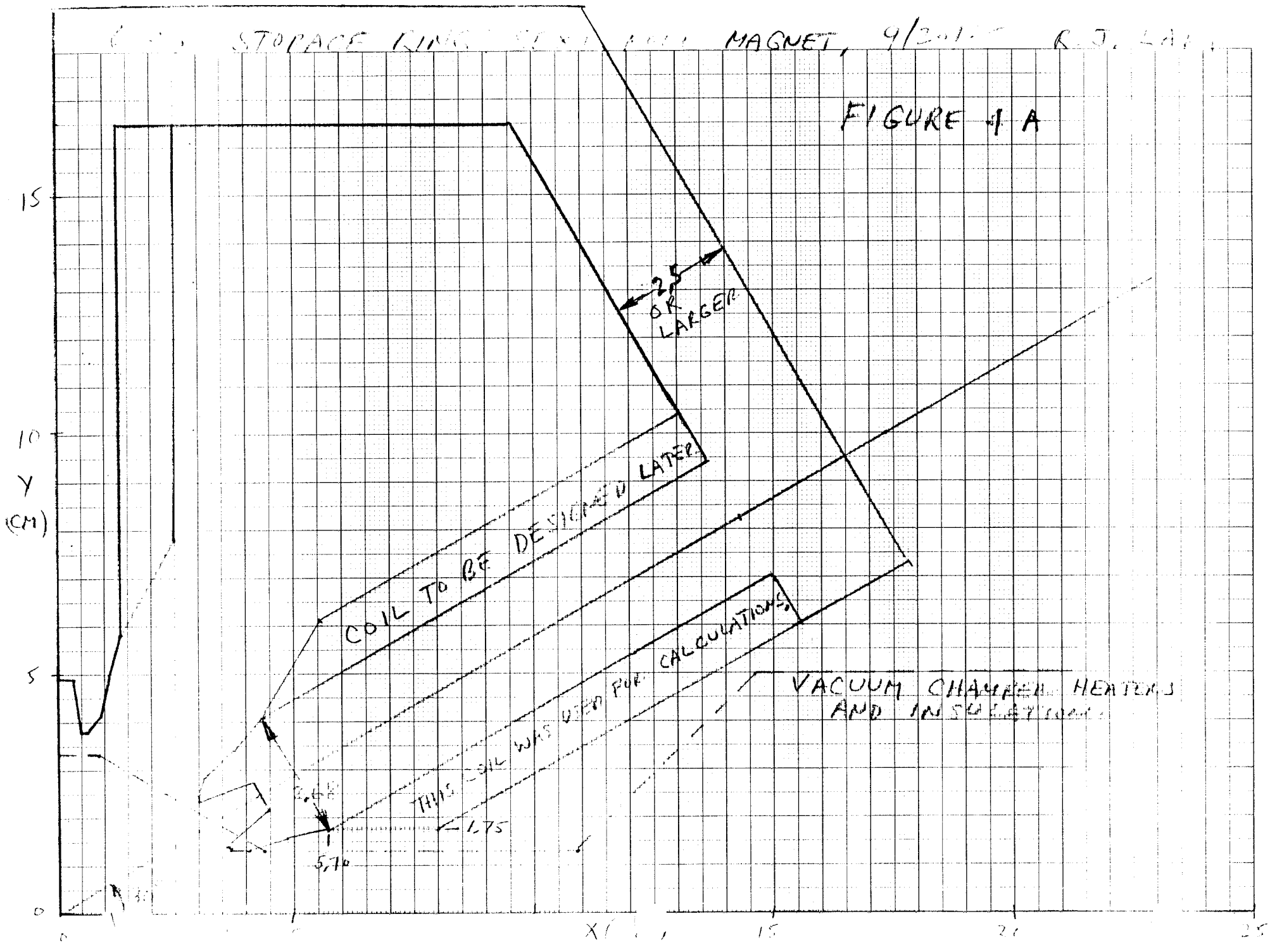

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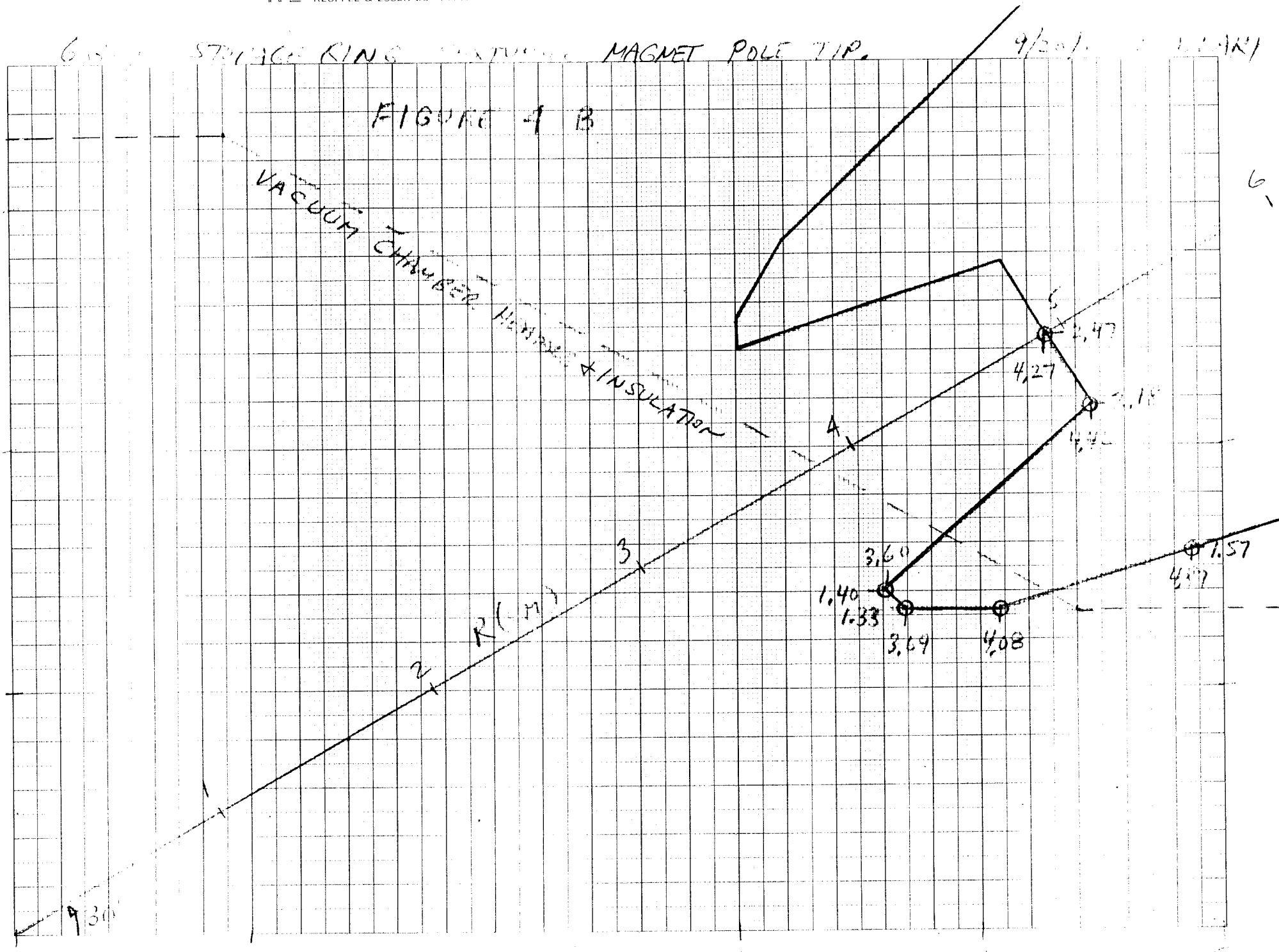

Figure 5. u-v Plane Flux Plot

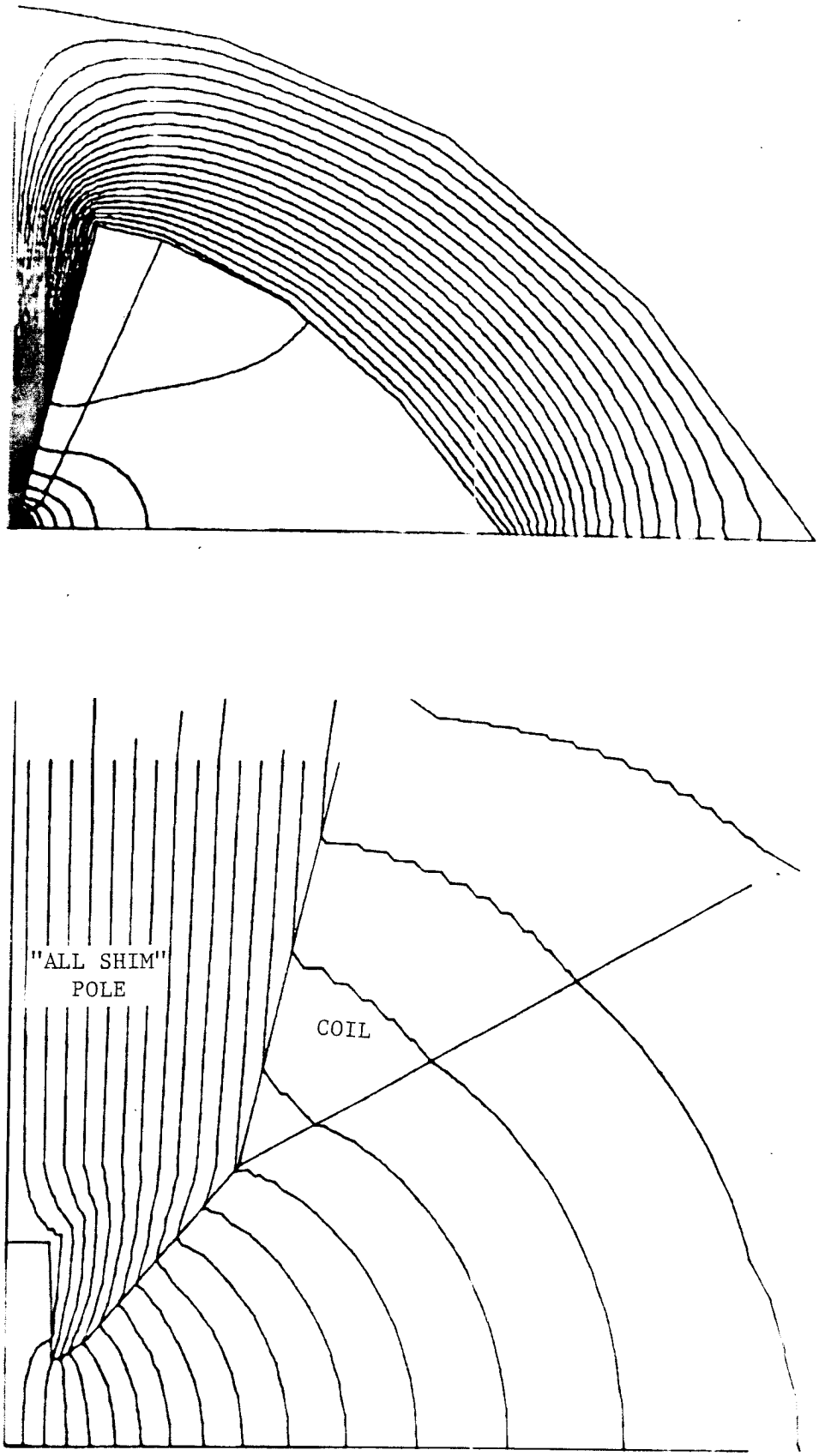

Table III

Sextupole Magnet

		UNSHIMMED MAGNET		ALL SHIM MAGNET				
Current		$\mu = \infty$	1010 Steel	$\mu = \infty$	1010 STEEL FULLY ANNEALED			
Multiplier		1.0	1.0	1.0	1.0	1.5	2.0	3.0
Field at R = 4 cm		2125.6	2102.0	2189.7	2164.1	3233.6	4075.8	4906.3
B_n Harmonic Fields at R = 3 cm (Gauss)	9	-80.3	-79.4	1.7	1.7	2.5	3.2	3.8
	15	-3.3	-3.2	-36.0	-35.5	-53.1	-66.9	-80.3
	21	0.8	0.8	5.6	5.5	8.2	10.4	12.4
	27	0.0	0.0	0.0	0.1	0.1	0.1	0.1
Efficiency		.949	.938	.978	.966	.962	.910	.730
Max Pole Field (kG)			9.4	--	9.6	14.4	18.0	21.3
Yoke Field (kG)		--	6.8	--	6.6	10.4	14.2	15.5
Stored Energy (J/M)		11.9	11.7	12.1	12.0	26.8	45.0	80.6
a_2(G/cm^2)		266.	263.	274.	271.	404.	509.	613.
a_8(G/cm^8)		-49.4	-48.8	1.0	1.0	1.5	2.0	2.3
a_{14}(G/cm^{14})		-1.1×10^3	-1.0×10^3	-1.2×10^4	-1.2×10^4	-1.7×10^4	-2.2×10^4	-2.6×10^4
a_{20}(G/cm^{20})		1.8×10^8	1.8×10^8	1.2×10^7	1.2×10^7	1.8×10^7	2.3×10^7	2.7×10^7