LS-46
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e$^-$ Linac - Axial Magnetic Field Confinement

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This is to determine the axial magnetic field in the first part of the e$^-$ Linac (up to 100 MeV) that are appropriate to contain the beam in the radial direction.

The difocussing field component of the traveling wave and the space change forces are to be included in the calculation. In cylindrical coordinates, Lorenzen eq.

$$\frac{d\vec{p}}{dt} = e(\vec{E} + \vec{V} \times \vec{B})$$

becomes:

$$\frac{d}{dt}(\gamma\dot{z}) = \frac{e}{m_o}(E_z + \dot{r}\beta_\phi - r\dot{\phi}B_r) \tag{1a}$$

$$\frac{d}{dt}(\gamma\dot{r}) = \frac{e}{m_o}(E_r + r\dot{\phi}B_z - \dot{z}B_\phi) + \gamma r\dot{\phi}^2 \tag{1b}$$

$$\frac{d}{dt}(\gamma r^2\dot{\phi}) = \frac{e}{m_o}(E_\phi + \dot{z}\ B_r - \dot{r}B_z) \tag{1c}$$

In (1c) B_ϕ and B_z are magnetic field components due to the applied stationary magnetic field.

Assume B_z is independent of r and there is no circumferential component of the field, then since:

$$\nabla \cdot B = \frac{1}{r}\frac{\partial}{\partial_r}(rB_r) + \frac{1}{\gamma}\frac{\partial B\phi}{\partial\phi} + \frac{\partial B_z}{\partial z} = 0$$

we put

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$

The physical meaning of this is that a change in the axial field can only come from the existence of the radial field thus (1c) can be written

$$\frac{d}{dt}(\gamma r^2\dot{\phi}) = -\frac{e}{2m_o}\frac{d}{dt}(r^2 B_z)$$

The solution of which is

$$\gamma r^2\dot{\phi} = -\frac{e}{2m_o} r^2 B_z + [\gamma_o r_o{}^2\dot{\phi}_o]$$

The last term indicates initial conditions.

In order to study the radial momentum equation we must specify the field components of the traveling wave.

In the region of beam interaction we have for the fundamental wave:

$$E_z = -E_o J_o(k_1 r)\sin\delta = \simeq E_o \sin\delta$$

$$E_r = E_o \frac{k_z}{k_1} J_1\ (k_1 r)\ \cos\delta \simeq E_o \frac{k_z r}{2} \cos\delta$$

$$B_\phi = \left(\frac{E_o}{c}\right) \frac{k}{k_1} J_1\ (k_1 r)\ \cos\delta \simeq \frac{E_o}{c}\ \frac{k\cdot r}{2} \cos\delta$$

Assuming that the particle does not stray far from the axis we may set

$$J_1\ (k_1 r) = k_1 \frac{r}{2} \text{ and } J_0\ (k_1 r) = 1$$

where k_z is a propagation constant and $k = 2\pi/\lambda$, $k_1 = k\left(\frac{\sqrt{\beta^2-1}}{\beta}\right) = 2\pi/\lambda\ \left(\frac{\sqrt{\beta^2-1}}{\beta}\right)$

(1b) can now be written

$$\frac{d}{dt}(\gamma\dot{r}) = \frac{e}{m_o} E_o \frac{\pi r}{\lambda} \left(\frac{1-\beta^2}{\beta}\right) \cos\delta - \gamma r \left(\frac{eB_z}{2\gamma m_o}\right)^2$$

Now B_z is chosen so that the radial momentum is constant:

$$\frac{d(\gamma\dot{r})}{dt} = 0$$

hence

$$B_z = \sqrt{\frac{4\pi m_o}{e\lambda} E_o \cos\delta \left(\frac{1-\beta^2}{\beta}\right)} \qquad \text{Tesla}$$

In order to include the effects of space change, we add in the radial equation of motion the relativistic self repulsive force. (For simplicity, we can use the expression for a continuous beam)

$$F_r = \frac{eIn}{2\pi} \left(\frac{1-\beta^2}{\beta}\right) \left(\frac{r}{\gamma_o^2}\right) \phi$$

where ϕ is the bunching factor and r_o the beam diameter. Now equation (1b) may be written:

$$\frac{d(\gamma\dot{r})}{dt} = \frac{e}{m_o} \left[E_o \frac{\pi r}{\lambda} \cos\delta + \frac{Inr\phi}{2\pi r_o^2}\right] \frac{1-\beta^2}{\beta} - \gamma r \left(\frac{eB_z}{2\gamma m_o}\right)^2$$

choosing B_z such that the radial momentum is constant $\frac{d}{dt}(\gamma\dot{r}) = 0$ and solving for B_z

$$B_z = \sqrt{\frac{4m_o}{e\beta\gamma} \left[E_o \frac{\pi}{\lambda} \cos\delta + \frac{In\phi}{2\pi ro^2}\right]} \qquad \text{Tesla}$$

$$B_z = \sqrt{\frac{4m_o}{e\beta\gamma}\left[E_o \frac{\pi}{\lambda} \cos\delta + \frac{I\phi}{2\pi r_o^2}\right]} \quad \text{Tesla}$$

B_z after the second prebuncher just before the main buncher.

$I = 30$ Amps

Voltage = 0.136 V

$\beta = 0.62$

$\gamma = 1.264$

$r_o = 3 \times 10^{-3}$

$\phi = 10$

$r_o = 3 \times 10^{-3}$

$\lambda = 0.1$ m

$E_o = 3 \times 10^6$

$$B_z = \sqrt{\frac{4 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.78}\left[3 \times \frac{10^6}{0.1}\pi 1 + \frac{30 \times 337}{2\pi \times 9 \times 10^{-6}}\right]10^6}$$

$$B_z = \sqrt{29.17 \times 10^{-12}\left[94.25 \times 10^6 + 200 \times 10^6\right]}$$

$$B_z = \sqrt{29.17 \times 294}$$

$$= 9.6 \times 10^{-3} \text{ Tesla}$$

$$= 926 \text{ Gauss}$$

If $\cos\delta = 0$ the beam is on the peak of the wave:

$$B_z = \sqrt{29.17 \times 200 \times 10^{-6}} = 76.3 \times 10^{-3} \text{ Tesla}$$

or 763 Gauss

B_z required at the buncher $-\phi = 30.$

$E_o = 10$ MeV/m $= 10^7$

Ave. energy of $e^+ = 1.25$ MeV

$\beta = 0.95$

$\gamma = 3.2$

$r_o = 3 \times 10^{-3}$

$$B_z = \sqrt{\frac{4m_o}{e\beta\gamma}\left[E_o \frac{\pi}{\lambda}\cos\delta + \frac{\text{In}\phi}{2\pi r_o^{\,2}}\right]} \quad \text{Tesla}$$

$$= \sqrt{\frac{4 \times 9.1 \times 10^{-31}\ 10^{-12}}{1.6 \times 10^{-19} \times 0.98 \times 3.2}}\ \left[\frac{10^7\pi}{0.1}\cos\delta + \frac{90 \times 377}{2\pi \times 9 \times 10^{-6}}\right]10^6$$

$\cos\delta = 1$

$$B_z = \sqrt{7.48 \times 10^{-12}[914.17 \times 10^6]}$$

$$= 82.7 \times 10^{-3} \text{ Tesla}$$

$$= 827 \text{ Gauss}$$

$\cos\delta = 0$

$$B_z = \sqrt{7.48 \times 10^{-12} \times 600.01 \times 10^6}$$

$$= 670 \text{ Gauss}$$