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TOUSCHEK LIFETIME CALCULATIONS

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The Touschek lifetime calculated by Bruck[1] Eq. 30.17

$$\frac{1}{\tau_T} = \frac{dN}{Ndt} = \frac{\sqrt{\pi}\, r_e^2\, cN}{\Delta P_x (\Delta P_{rf})^2 V_p} \quad C(\varepsilon) = \frac{N}{\alpha} \tag{1}$$

is proportional to N(t). Therefore, the functional form of N(t) is not exponential, but rather given by[2]

$$N(t) = \frac{N_o}{1 + \frac{N_o t}{\alpha}}$$

where $N_o = N(t = 0)$ the number of particles in the bunch at time t = 0.

Since τ_T is usually calculated for $N = N_o$

then
$$N(t) = \frac{N_o}{1 + \frac{t}{\tau_T}} ,$$

At time $t = \tau_T$, the number of particles in the bunch is $N(t = \tau_T) = 1/2\ N_o$, i.e., τ_T is a half lifetime. Then the time to decay to e^{-1} is $\tau_e = 1.7183\ \tau_T$. Care must be taken in using Eq. (1) because of the normalized units used for ΔP_x and ΔP_{rf}

$$\Delta P_x = \gamma\ \sigma_x'$$

and
$$\Delta P_{rf} = \gamma\left(\frac{\delta p_{rf}}{p_o}\right)$$

where $\sigma_x' = \sqrt{\frac{\varepsilon_x}{\beta_x}}$, δp_{rf} = rf momentum bucket height for synchronous momentum p_o

and γ = relativistic gamma $= \frac{1}{\sqrt{1-\beta^2}}$.

A simpler expression for Eq. (1) is

$$\frac{1}{\tau_T} = \frac{\sqrt{\pi}\, r_e^{\,2}\, c N_o}{\gamma^3\, \sigma_x' \left(\frac{\delta p_{rf}}{p}\right)^2 V_p}\, C(\varepsilon) \tag{2}$$

where $\varepsilon = \left(\frac{\delta p_{rf}}{p_o \gamma \sigma_x'}\right)$, $C(\varepsilon) = \ell n\left(\frac{0.5618}{\varepsilon}\right) - 1.5$ and $V_p = (4\pi)^{3/2}\, \sigma_x \sigma_y \sigma_\ell$.

The program ZAP[3] and BEAMPARAM[4] calculate $1/\tau_T$ by averaging the rate over the lattice functions. BEAMPARAM doesn't allow for momentum apertures other than rf bucket heights (δp_{rf}) and calculates τ_T only for the zero current bunch length $\left[\sigma_\ell = \frac{\alpha c}{2\pi f_s}\left(\frac{\sigma_E}{E_o}\right)\right]$ ZAP allows an arbitrary σ_ℓ and also δp physical apertures for both dispersive and non-dispersive sections of the lattice. To compare the difference in τ_T for these different calculation methods, the CDR lattice for the 6-GeV light source was used with a momentum aperture set by an rf bucket height of 1.98%. The resulting τ_T from BEAMPARAM and ZAP with the zero current bunch length of σ_ℓ = 0.62 cm are plotted in Figure 1. Agreement is typically within 2%. Also shown are τ_T calculated by ZAP for bunch lengths of σ_ℓ = 1 and 2 cm. Two data points (labeled Bruck) are plotted in Figure 1, which are the result of calculations using Eq. (2) with σ_ℓ = 1 cm and the average values of β_x and β_y for the lattice. These data are 10 to 15% greater than the values calculated by ZAP, which averages the loss rate over the lattice.

Figure 1 indicates an expected increase in lifetime as the bunch lengthens, and the energy spread increases. This is primarily an effect of the decrease in charge density within the bunch which causes a decrease in the collision rate. However, the Bruck theory doesn't integrate over the momentum spread of the bunch and therefore doesn't account for an increased loss of particles closer to the rf separatrix. This effect is expected to be a small correction to the theory.

References

1. H. Bruck, Accélérateurs Circulaires de Particles, LANL Report, LA-TR-72-10, translated by R. McElroy Co. Inc. (1972).

2. S. Krinsky, "Handbook on Synchrotron Radiation" edited by E. E. Koch, North-Holland Publishing Co., Vol. 1A, pp. 120-122 (1983).

3. J. Bisognano et al., IEEE NS-32, 2700 (1985).

4. M. Gygi-Hanney et al., CERN/LEP-TH/83-2 (1983).

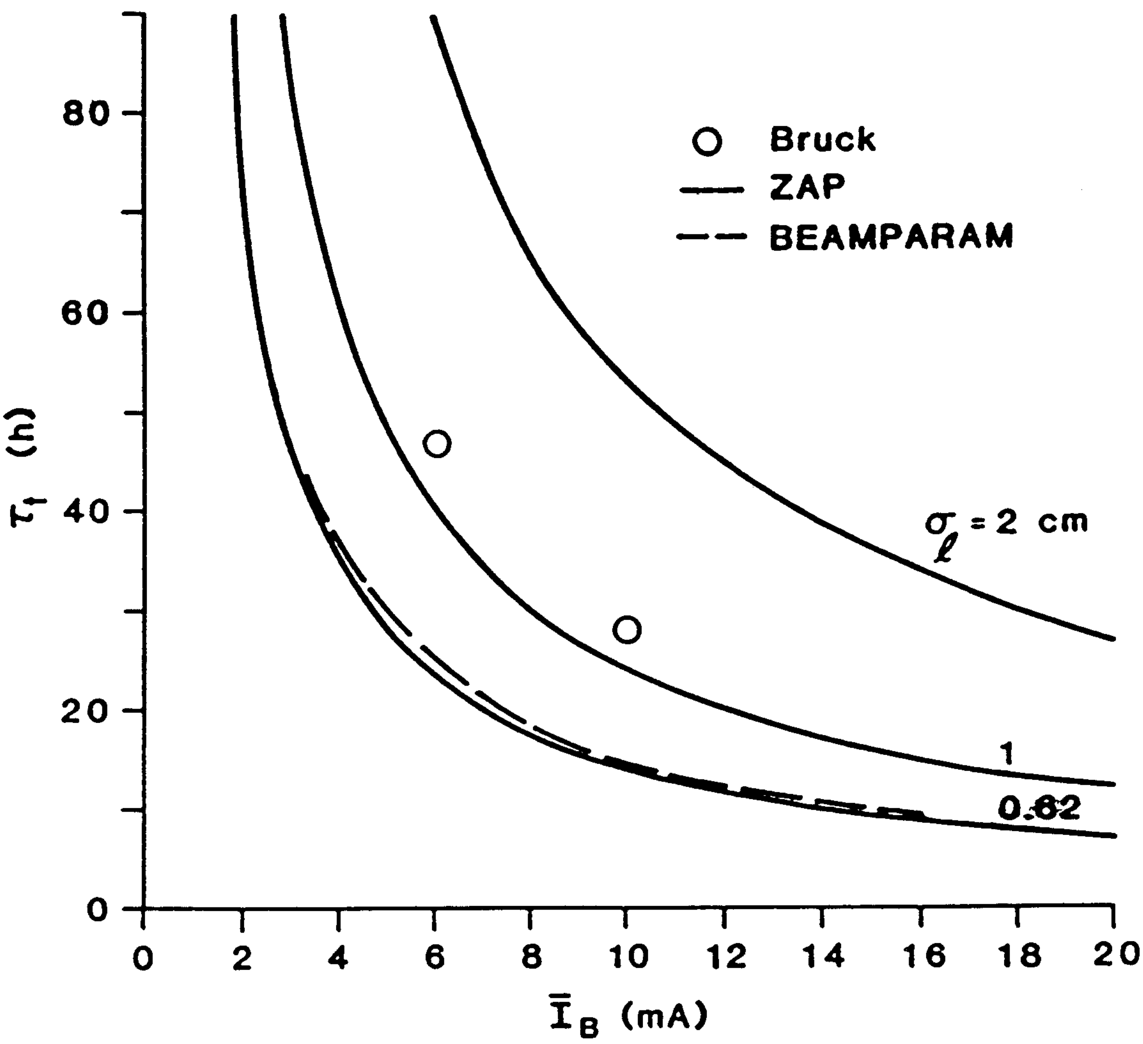

Figure 1. Touschek lifetimes as a function of average bunch current calculated for the CDR 6 GeV light source lattice with $\delta p_{rf}/p = \pm 1.98\%$ and zero current bunch length of σ_ℓ = 0.62 cm ($\sigma_E/E_o = \pm 0.1\%$). The solid curves are the results from the program ZAP with σ_ℓ = 0.62, and 1 and 2 cm. The dashed curve is the result from the program BEAMPARAM with σ_ℓ = 0.62 cm. The two data points are from the calculations using Eq. (2) (Bruck Eq. 30.17) with σ_ℓ = 1 cm and the average β_x and β_y for the CDR lattice.